EIGENVALUES EMBEDDED IN THE CONTINUUM FOR NEGATIVELY CURVED MANIFOLDS

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1. INTRODUCTION

Suppose that M is a complete simply connected negatively curved surface and Δ is the Laplacian of M. If M is the Poincaré upper half plane with constant curvature -1, then the spectrum of $-\Delta$ is purely continuous and consists of the half line $[1/4,\infty)$.

Denote K to be the Gauss curvature of M. McKean [10] showed that if $K \leqslant -1$ then the spectrum of $-\Delta$ is bounded below by $1/4$. By a more detailed argument, Pinsky [14] proved that if $K \leqslant -1$ then $1/4$ does not appear in the point spectrum of $-\Delta$.

Since M may have continuous spectrum starting at $1/4$, new proofs are required to prevent M from having eigenvalues greater than $1/4$. Let $ds^2 = dr^2 + g^2(r,\theta)\,d\theta^2$ be the metric in terms of geodesic polar coordinates (r,θ) about some $p \in M$. If $g = g(r)$ is independent of θ, then Pinsky [14] gave decay conditions on $K(r) + 1$, as $r \to \infty$, which insure that M has no eigenvalues greater than $1/4$. Unfortunately, his method does not generalize in a straightforward way to metrics which are not rotation invariant, at least for r suitably large.

In this paper we give decay conditions on $K(r,\theta) + 1$, K_θ, $K_{\theta\theta}$, as $r \to \infty$, which imply that M has no eigenvalues greater than $1/4$. We then easily generalize our results to dimensions $n \geqslant 2$. By adding the hypothesis $K \leqslant -1$, one obtains a criterion for a negatively curved manifold to have purely continuous spectrum consisting of the half line $[(n-1)^2/4,\infty)$.

Our method is a modification of Kato's solution [9] given the analogous problem for the Schrödinger operator on $\mathbf{R}^n$. The idea is to regard $L^2(\mathbf{R}^n) = L^2(\mathbf{R}) \times L^2(S^{n-1})$ and to systematically exploit differential inequalities for $L^2(S^{n-1})$ valued functions on $\mathbf{R}$.

The author thanks Professor Pinsky for sending us a copy of his paper [14] and for informing us [12] of the open problem which arose in that work. This provided the starting point for the present paper.

2. SURFACES HAVING ASYMPTOTICALLY CONSTANT CURVATURE

Let M be a complete simply connected negatively curved surface. Then for each $p \in M$ the exponential map exp: $T_pM \to M$ is a diffeomorphism [3, p. 184].

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