THE THEORY OF MOTION GROUPS

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INTRODUCTION

In 1962 in his Ph.D. thesis, David M. Dahm defined the group of motions with compact support of the compact subspace N in the manifold M to be the group of essentially different ways of continuously moving N in M, so that at the end of the motion, N has been returned to its original position. This paper is mostly my exposition of Dahm's unpublished work.

It was the idea of Hurwitz (see [18]), and later of Fox (see [12]), to envisage a braid (see Figure 1) as a continuous 1-parameter family of changing configurations of n distinct points in the xy-plane, where at each time t_0, the configuration is given by the intersection of the braid with the plane at height $z = t_0$. Thus, the motion group $\mathscr{M}(M,N)$ has its origins in the Artin braid group ([1], [2], [3] and [4]).

The sections of this paper are

1. The Braid Groups
2. Motion Groups Defined
3. Properties of Motion Groups
4. The Dahm Homomorphism
5. The Group of Motions of a Collection of n Unknotted, Unlinked Circles in $\mathbf{R}^3$

The background and motivation of the braid groups is discussed in Section 1. In Section 2 I define and give examples of motion groups. Section 3 is mostly a list of short exact sequences containing the group of motions as a term, from which this group may be computed. Section 4 defines a homomorphism

$$D: \mathscr{M}(M,N) \rightarrow \mathrm{Aut}\,(\pi_1(M - N))$$

from the group of motions of N in M, to the automorphisms of $\pi_1(M - N)$ induced at the end of the motion.

The main result of the paper, in Section 5, is the following calculation: Let $C = C_1 \cup \ldots \cup C_n \subset \mathbf{R}^3$ be a collection of n unknotted, unlinked circles in $\mathbf{R}^3$. Let $F(x_1, \ldots, x_n)$ denote the free group on n generators, x_i, $i = 1, \ldots, n$; note that $\pi_1(\mathbf{R}^3 - C) \simeq F(x_1, \ldots, x_n)$.

THEOREM 5.4. *The group of motions* $\mathscr{M}(\mathbf{R}^3, C)$ *of the trivial n-component link* C *in* $\mathbf{R}^3$ *is generated by the following types of motions:*

Received December 14, 1977. Revision received October 5, 1978.

Michigan Math. J. 28 (1981).