SPLITTING THE PL INVOLUTIONS OF NONPRIME 3-MANIFOLDS

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INTRODUCTION

We describe four basic operations (I-operations) involving the connected sum construction with which PL involutions of 3-manifolds can be built up from involutions of simpler 3-manifolds. The main result (Theorem 1) is that every PL involution of a compact 3-manifold arises from involutions on its prime summands by repeated application of these four I-operations. It is well-known that every compact 3-manifold can be uniquely expressed (up to order) as the connected sum of prime 3-manifolds in normal form. Thus the study of PL involutions of compact 3-manifold is now reduced to problems involving PL involutions of prime 3-manifolds.

Section 1 is devoted to the descriptions of the I-operations and stating the main results. An application of Theorem 1 to double-coverings of S^3 branched over a link is also given here. Theorem 1 has also been applied to $P^3 \# P^3$ to show that there exist exactly seven distinct nonconjugate involutions on $P^3 \# P^3$ (see [5]). Section 2 contains the proof of Theorem 1. Finally, in Section 3, we prove a basic lemma for splitting 3-manifolds with involution along disks and suggest a further reduction for PL involutions of compact irreducible 3-manifolds with boundary with respect to the multi-disk sum operation.

1. STATEMENT OF RESULTS

We work exclusively in the PL category throughout this paper. All orientable 3-manifolds are assumed to be oriented. We let M^- denote the 3-manifold obtained from an oriented 3-manifold $M = M^+$ by reversing its orientation. Recall that the *connected sum* $M_1 \# M_2$ of two connected 3-manifolds M_1 and M_2 is obtained by removing the interior of a closed 3-cell from the interior of each and identifying the resulting 2-sphere boundaries by a homeomorphism (orientation reversing if both M_1 and M_2 are oriented). A compact 3-manifold M is said to be *prime* if it cannot be written as a connected sum of two 3-manifolds, each distinct from S^3. Recall that S^3 is the identity element for this operation. According to the unique decomposition theorem of Kneser [7] and Milnor [11] (see Hempel [4]), every compact 3-manifold can be written uniquely (up to order) as a connected sum of prime 3-manifolds in normal form (in the normal form $S^1 \times S^2$ is allowed to appear as a summand only when M is orientable). It follows that a compact 3-manifold can be built up in an essentially unique way from prime 3-manifolds.

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