ON MEROMORPHIC SOLUTIONS OF A LINEAR DIFFERENTIAL-DIFFERENCE EQUATION WITH CONSTANT COEFFICIENTS

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INTRODUCTION

A. O. Gelfond considered in his paper [4], among other questions, the entire solutions of the double system of equations (S) $L[F(z)] = 0$, $M[F(z)] = 0$, where

$$L[F(Z)] = \sum_{k=1}^{m} c_k F(z+\gamma_k), \qquad M[F(z)] = \sum_{k=1}^{n} d_k F(z+\delta_k)$$

are linear difference operators with constant coefficients c_k, d_k and steps γ_k, δ_k. If $L[F(z)] = F(z+\alpha) - F(z)$, $M[F(z)] = F(z+\beta) - F(z)$, $\operatorname{Im}(\alpha/\beta) \neq 0$, then the meromorphic solutions of the system (S) are the much studied elliptic functions. Gelfond's result on the entire solutions of the general system (S) is, for this particular case, identical with the well known theorem which states that there are no entire elliptic functions other than $f(z) \equiv$ constant.

A. I. Markushevich suggested in connection with Gelfond's paper the following question: What can be said about the meromorphic solutions of the double system (S)?

The meromorphic solutions of the system (S), as well as of other systems (not necessarily double systems) of difference and differential-difference equations, were afterwards studied by the authors of this paper, by L. Navickaite, R. Sandler, T. S. Silver, V. Tevelis, and L. Trushina. (see [7], [8], [14]-[34], [37], [38]). Earlier material on meromorphic solutions of difference equations may be found in the monograph [1] of P. Appell and E. Lacour and in the monograph [35] of E. Picard. This subject has also been studied in the papers [2]-[6], [9]-[13], [39] of F. Erwe, G. Floquet, M. Ghermanesco, A. Hurwitz, H. Löwig, P. Montel, and J. M. Whittaker.

In this paper we consider the meromorphic solutions $f(z)$ of the differential-difference equation

(1) $$A[f(z)] \equiv A_0[f(z)] + A_1[f'(z)] + A_2[f''(z)] + \dots + A_n[f^{(n)}(z)] = 0,$$

where

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