QUASICONFORMAL VARIATION OF THE GREEN'S FUNCTION

Ignacio Guerrero

In this paper we study the variation of the Green's function of a Riemann surface due to a perturbation of the conformal structure. The perturbations considered are parametrized by the complex dilatation of a quasiconformal map.

The basic idea in our approach is to consider the Green's function of a Fuchsian group. Results concerning Riemann surfaces are obtained via uniformization. The advantage of this point of view is that we can make use of powerful results about normalized quasiconformal maps of the unit disc. The main problem becomes to justify the validity of term by term estimates in the series defining the Green's function.

Variations of domain functionals, in particular variations of the Green's function, have been extensively studied. For previous results we refer to Sontag [7] and the references listed there.

Section 1 contains some basic results and definitions used in the paper. In section 2 we treat the Fuchsian group case. Section 3 gives applications to Riemann surfaces.

Results in this paper are based on part of the author's 1975 Stony Brook dissertation.

Finally, we would like to point out that we do not know how to generalize the main varational formula for infinitely generated groups or, equivalently, for arbitrary (not necessarily finite) hyperbolic Riemann surfaces.

1. PRELIMINARIES

1.1 A Fuchsian group Γ acting on the unit disc U is said to be of convergence type if

$$\sum_{\gamma \in \Gamma} (1 - |\gamma(z)|) < \infty, \tag{1.1}$$

uniformly for z in a compact subset of U. This condition is equivalent (Tsuji [8, p. 522]) to the existence of a Green's function on the Riemann surface U/Γ. Actually, we can write explicitly the Green's function as a function on U invariant under Γ. Namely

$$G(z, a) = \sum_{\gamma} \log \left| \frac{z - \gamma(a)}{1 - \overline{\gamma(a)}\, z} \right|. \tag{1.2}$$

Received May 30, 1978.

Michigan Math. J. 26 (1979).