FREE HEEGAARD DIAGRAMS AND EXTENDED NIELSEN TRANSFORMATIONS, I

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This is the first of two papers (see also [7]) devoted to the study of a free analogue, called here a free splitting homomorphism, of the algebraic formalism for Heegaard splittings of 3-manifolds due to Stallings [30] and Jaco [10]. Roughly, free splitting homomorphisms come from replacing surface groups by free groups in the Stallings-Jaco formalism. A free splitting homomorphism has the form, $\psi = \psi_1 \times \psi_2 \colon G^m \to X^n \times Y^n$, where G^m, X^n, and Y^n are free groups of ranks m, n, and n respectively, and each of the factor homomorphisms ψ_1 and ψ_2 is surjective. We will show in Theorem 4.1 that, after allowances are made for stabilization, the free splitting homomorphism theory is equivalent to the theory of extended Nielsen transformations [2]. (Extended Nielsen transformations will be described in Section 1.)

The connection between the two theories will be made by normalizing a free splitting homomorphism ψ as above so that for free bases $\{g_i : i \leq m\}$ and $\{x_i : i \leq n\}$ for G^m and X^n, ψ has the form $\psi(g_i) = (x_i, v_i)$ $(i \leq n)$ and $\psi(g_{i+n}) = (1, r_i)$. One will then have an associated group presentation $\mathscr{P}(\psi) = \langle Y^n : (r_i) \rangle$. Theorem 4.1 states that two free splitting homomorphisms ψ and ϕ are stably equivalent if and only if, after normalization, the associated group presentations $\mathscr{P}(\psi)$ and $\mathscr{P}(\phi)$ are equivalent in the sense of extended Nielsen transformations including stabilization.

Two applications of Theorem 4.1 will then be given. The first of these, Theorem 5.2, concerns simplifying a group presentation $\langle Y^n : (r_i) \rangle$ to a presentation $\langle Y^q : (s_i) \rangle$ $(q < n)$ through the use of extended Nielsen transformations including stabilization. Theorem 5.2 says that this can be done if there are q elements $w_1, \ldots, w_q$ in Y^n such that $\{w_i\} \cup \{r_i\}$ generates Y^n. The second application, Theorem 5.3, shows that the stable form of the Andrews-Curtis conjecture on presentations for the trivial group holds if and only if, in a stable sense, the conclusion of the Grusko-Neumann theorem (see [9], [22], [11], [29], [12], [17]) holds for surjective homomorphisms of the form, $G^{2n} \to X^n \times Y^n$ where G^{2n}, X^n, and Y^n are free groups as before.

The second of the two papers will apply methods due to Rapaport [26] to compare equivalence for free splitting homomorphisms with equivalence under extended Nielsen transformations. Another criterion for simplifying a presentation will be given which mostly involves normal closure properties. Finally, some examples will be proposed which we suspect distinguish between two different kinds of extended Nielsen equivalence classes for balanced presentations of the trivial group.

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