# FUNCTIONAL INTEGRALS RELATED TO A NONCONTRACTION SEMIGROUP

T. F. Lin

## 1. INTRODUCTION

Since the controversial Feynman path integral (see [3,6]) appeared, functional integral representations for solutions of certain initial value problem $u_t = Au + vu$ have been extensively studied. There are essentially two types of integral representations. In one type of representation, $A$ is the infinitesimal generator of a contraction semigroup $S_t$ such that $S_t 1 = 1$, $\|S_t\| = 1$. The measure involved in the integration is a probability measure which is associated with a diffusion process (see [5,1]). For generalization to the nonhomogeneous case, see [10]. In other types of representations, $A$ is associated with a semigroup of operators $S_t$ such that $S_t 1 = 1$ and $\|S_t\| \equiv c > 1$. The integration is carried out with respect to a finitely additive set function (see [7,2]). We will study here a type where $A$ generates a semigroup of operators $S_t$ such that $S_t 1 = 1$ and $\|S_t\| \leq e^{\alpha t}$ for some $\alpha \in R^+$. The measure to be used in the integration is a measure, perhaps complex or signed, with total measure 1. Note that the condition $S_t 1 = 1$ is indispensable in the construction of measures or set functions on function spaces. The difference between the above three cases is the norm of $S_t$.

Throughout this article, unless otherwise specified, $X$ denotes a compact metric space with metric $\rho$, $C = C(X)$ denotes the space of all real continuous functions on $X$ with supremum norm and $A$ denotes a closed linear operator on $C$ with domain $\mathscr{D}(A)$ dense in $C$ and containing all constant functions.

It will be shown that if $A$ satisfies the following conditions:

(1.1) $$Af(x_0) \leq \alpha f(x_0) \text{ if } f(x_0) = \|f\|, \text{ where } \alpha \in R^+,$$

(1.2) $$\lambda - A \text{ maps } \mathscr{D}(A) \text{ onto } C \text{ for each } \lambda > \alpha,$$

(1.3) $$A1 = 0,$$

then the solution of the initial value problem

(1.4) $$\begin{cases} u_t(t,x) = Au(t,x) + v(x)u(t,x), \\ u(0,x) = f(x), \end{cases}$$

$0 \leq t \leq T < \infty$, $v \in C$, $f \in \mathscr{D}(A)$, has a functional integral representation. The solution can also be expressed as

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