AN ALGORITHM FOR HALTER-KOCH UNITS

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Dedicated to James M. Vaughn, Jr.

INTRODUCTION

In a most remarkable paper Halter-Koch [6] gives the following abstract of his results (translated from German): "By means of the Modified Jacobi-Perron Algorithm, Bernstein, Hasse and Stender have constructed a system of independent units for some (infinite) classes of totally real algebraic number fields; they have shown that in certain cases (for infinitely many algebraic fields of degree $n = 3, 4, 6$) this system represents a basis of fundamental units or how such a basis can be obtained from the system. In the present paper I shall disclose maximal independent systems of units for a much wider class of algebraic number fields. Whether these units can again be obtained from an algorithm, and in which cases they turn out to be fundamental, is subject to further investigations." (The remarks in parentheses were added by the author.)

In this paper the author will solve the first of the two challenging problems posed by Halter-Koch: he will construct an algorithm by means of which he will obtain all the units found by Halter-Koch.

It must be emphatically mentioned that the Jacobi-Perron [7], [8] algorithm or its modification as used by the author [2] are especially significant when they become periodic. The author succeeded in disclosing a few classes of infinitely many algebraic number fields of any degree $n \geq 2$, a proper basis of which becomes periodic by the Jacobi-Perron algorithm or its generalization [1], [2]; on the basis of an important theorem by Hasse and the author [5], units from these algorithms which were in some cases a maximal independent system of units; Stender [9] has proved that for these fields of degree $n = 3, 4, 6$ this forms a system of fundamental units. The author will prove that the above theorem for units holds also for the new algorithm exposed in the next chapter.

1. THE ZERO ALGORITHM

Though this differs in its structure from the Jacobi-Perron Algorithm [7], [8] and the Modified Jacobi-Perron Algorithm, as introduced by the author [1], [2], it preserves the same basic properties which will therefore be enumerated, but not proved here. Their proof is exclusively based on induction.

Definition. Let $F(x)$ be an n-th degree polynomial in one variable x over the field A of algebraic numbers, *viz.*

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