PSEUDO-LINEAR SPHERES

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INTRODUCTION

This work has been motivated by an analysis made on the pseudo-free actions constructed by Montgomery and Yang [10]. More generally, we study pseudo-linear S^1 spheres; that is, circle actions on cohomology spheres such that the fixed point set associated with each subgroup of S^1 is again a cohomology sphere.

Section 1 provides our main tool which is a refinement of a theorem of Atiyah and Segal. This result is interesting in its own right. It essentially says that $K^1(X \times_G EG) = 0$ for a compact connected Lie group G acting on a space X which has $K^1(X) = 0$. This generalizes the well known fact that $K^1(BG) = 0$. In sections 2 and 3 we define an equivariant Euler characteristic geometrically on the pseudo-linear spheres and show that this class determines, and is determined by the algebraic and homotopical structure of these spaces. Theorem 2.5 is the central result that allows one to work with this invariant. We show in particular that a nontrivial S^1 map between pseudo-linear spheres implies divisibility of the Euler characteristics and this in turn has immediate geometrical consequences. In section 4 we show that twice the tangent bundle of a smooth pseudo-linear sphere is equivariantly stably trivial. Also, as a consequence of the algebraic machinery developed, we prove in section 5 a conjecture of Ted Petrie on homotopy complex projective spaces in a restricted situation [11].

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1. COMPLETION

Let G be a compact Lie group and X a locally compact G space. $K_G^*(X)$ denotes complex equivariant K-theory with compact support and $R(G) = K_G^0(\text{point})$ is the complex representation ring of G [15].

Atiyah and Segal [3,(5.1)] proved that if $K_G^*(X)$ is finitely generated over R(G) and $K^*(X) = 0$ then $\hat{K}_G^*(X) = 0$, where $\wedge$ denotes completion with respect to the ideal I_G of all elements in R(G) of virtual dimension zero:

$$\hat{K}_G^*(X) = \lim_{\substack{\leftarrow \\ n}} \frac{K_G^*(X)}{(I_G)^n K_G^*(X)}$$

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