q-PLURISUBHARMONIC FUNCTIONS AND A GENERALIZED DIRICHLET PROBLEM

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1. INTRODUCTION

Let Ω be a bounded domain in $\mathbb{C}^n$ and let q be an integer, $0 \leq q \leq n-1$. A C^2 function u defined in Ω is a q-plurisubharmonic function in Ω if its complex Hessian has $(n-q)$ nonnegative eigenvalues at each point of Ω. An obvious question is whether there is a definition for q-plurisubharmonic functions which are not necessarily C^2. Recall that an upper semicontinuous function defined in Ω is plurisubharmonic there if it is essentially subharmonic in every complex direction (see [6]). Thus the definition of plurisubharmonic function is reduced to a 1-complex dimensional definition, and the same is true for plurisuperharmonic functions. We give definitions of q-plurisubharmonic (and q-plurisuperharmonic) functions in Ω, with 0-plurisubharmonic and plurisubharmonic being equivalent. These definitions seem to be very natural for $\mathbb{C}^n$, are invariant under biholomorphic coordinate changes on $\mathbb{C}^n$, and are equivalent to those mentioned above if a function is actually C^2.

Let D be a bounded strictly pseudoconvex domain in $\mathbb{C}^n$, $n > 1$, with C^2 boundary, and suppose that b is a continuous real valued function defined on ∂D. We can solve the Dirichlet problem to find a harmonic function in D which assumes the given boundary values. The problem with this solution is that it is not invariant under biholomorphic coordinate changes on $\mathbb{C}^n$. In order to remedy this, Bremermann [3] considered the class of all continuous plurisubharmonic functions in D which are less than or equal to b on ∂D and applied Perron's method showing that the upper envelope $\bar{u}$ of this class exists and takes on the given boundary values. His solution $\bar{u}$ is plurisubharmonic in D, invariant under biholomorphic coordinate changes on $\mathbb{C}^n$, and if C^2, satisfies the homogeneous complex Monge-Ampere equation

$$[\partial\bar{\partial}u]^n = \underbrace{\partial\bar{\partial}u \wedge \ldots \wedge \partial\bar{\partial}u}_{n \text{ times}} = 0$$

in D. Later, Walsh [8] showed that $\bar{u}$ is continuous, and Bedford and Taylor [2] proved that $\bar{u}$ satisfied their distributional definition of the homogeneous complex Monge-Ampere equation.

Let D be a strictly q-pseudoconvex domain in $\mathbb{C}^n$ with C^2 boundary, and let b be a continuous function on ∂D. We prove that the upper envelope of all upper semicontinuous functions on $\bar{D}$ which are q-plurisubharmonic in D and less than

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