A GROWTH CONDITION FOR CLASS $\mathscr{A}$

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The original notes of this material and preliminary work in several other topics are being held in the Gerald R. MacLane collection of the Mathematics Library at Purdue University.

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1. INTRODUCTION

Some years ago I introduced the class $\mathscr{A}$ of functions $f(z)$ which are holomorphic in the unit disc $D = \{|z| < 1\}$ and possess asymptotic values at a dense set on ∂D ([8]). Let $M(r) = \max_\theta |f(re^{i\theta})|$. Then I proved ([8, Theorem 14]) that $f \in \mathscr{A}$ if

$$(1.1) \qquad \int_0^1 (1-r)\log M(r)\,dr < \infty.$$

More generally, $f \in \mathscr{A}$ if there exists a dense set Θ in $[0,2\pi)$ such that

$$(1.2) \qquad \int_0^1 (1-r)\log^+ |f(re^{i\theta})|\,dr < \infty \qquad (\theta \in \Theta),$$

and the arguments of [8] use only (1.2); note that (1.2) is compatible with arbitrarily large $M(r)$.

Several years later, R. Hornblower [5] significantly improved the condition (1.1) by proving that $f \in \mathscr{A}$ if

$$(1.3) \qquad \int_0^1 \log^+ \log^+ M(r)\,dr < \infty.$$

Further, Hornblower showed that (1.3) is almost sharp in the sense that there exist functions not in $\mathscr{A}$ for which, corresponding to each $\varepsilon > 0$,

$$\log^+ \log^+ M(r) < \frac{\varepsilon}{(1-r)\log \dfrac{1}{1-r}} \qquad (r_\varepsilon < r < 1).$$

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