EXTREME POINTS OF THE UNIT BALL OF THE BLOCH SPACE $\mathscr{B}_0$

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1. INTRODUCTION

Let Δ denote the open unit disc in the complex plane $\mathbb{C}$, and let Γ denote the boundary of Δ. If f is a function holomorphic in Δ, define M(f) by

$$M(f) = \sup \{|f'(z)|(1 - |z|^2) : z \in \Delta\}.$$

The *Bloch space* $\mathscr{B}$ consists of those holomorphic functions f for which M(f) is finite. The norm $||f|| = |f(0)| + M(f)$ makes $\mathscr{B}$ a Banach space. The set of f in $\mathscr{B}$ for which $\lim_{|z| \to 1} |f'(z)|(1 - |z|^2) = 0$ is a closed subspace of $\mathscr{B}$, denoted by $\mathscr{B}_0$. There are several characterizations of the functions in the Bloch space, and we refer the reader to [1], [2], [4], and [5]. The dual space of $\mathscr{B}_0$ is linearly homeomorphic with a Banach space I of functions holomorphic on Δ [1]. In fact,

$$I = \left\{ g : \int_0^1 \int_0^{2\pi} |g'(re^{i\theta})| \, r \, dr \, d\theta < \infty \right\}.$$

Further, the second dual of $\mathscr{B}_0$ is isometrically isomorphic to $\mathscr{B}$. Alaoglu's Theorem and the Krein-Milman Theorem then imply that the unit ball of $\mathscr{B}$ has extreme points. We show that the unit ball of $\mathscr{B}_0$ also has extreme points. The principal result of this paper is a characterization of the extreme points of the unit ball of $\mathscr{B}_0$.

We list here a theorem which plays a fundamental role in later proofs.

THEOREM A. *Let* G(x, y) *be a convergent real power series such that* G(0,0) = 0 *and* $G(0, y) = \sum_{n=s}^{\infty} b_n y^n$, *where* $s \geq 1$ *and* $b_s \neq 0$. *Then there are power series* $\Omega(x, y)$, $A_i(x)$ $(i = 0, 1, \ldots, s-1)$ *such that*

$$G(x, y) = (y^s + A_{s-1}(x) y^{s-1} + \ldots + A_0(x)) \, \Omega(x, y),$$

and $\Omega(0,0) \neq 0$.

Theorem A is a special case of the real analytic version of the Weierstrass Preparation Theorem (*cf., e.g.,* [7, p. 145]). A C^∞ version of this result (the Malgrange-Mather Theorem) can be found in [6, p. 94].

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