OPERATORS OF CLASS C_{00} OVER MULTIPLY-CONNECTED DOMAINS

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INTRODUCTION

Let R be a domain in the complex plane bounded by $n + 1$ nonintersecting analytic Jordan curves, let $C(\partial R)$ be the space of continuous functions on the boundary of R, and let $\mathrm{Rat}(\bar{R})$ be the uniform closure in $C(\partial R)$ of the space of rational functions with poles off of $\bar{R}$. Let $\mathscr{H}$ be a complex Hilbert space and let $\mathscr{L}(\mathscr{H})$ be the algebra of bounded linear operators on $\mathscr{H}$. M. B. Abrahamse and R. G. Douglas [4] have recently initiated the study of contractive unital $\mathscr{L}(\mathscr{H})$-valued representations of $\mathrm{Rat}(\bar{R})$; that is, algebra homomorphisms

$$\sigma: \mathrm{Rat}(\bar{R}) \to \mathscr{L}(\mathscr{H})$$

such that $\|\sigma(f)\| \le \|f\|$ and $\sigma(1) = I_{\mathscr{H}}$. The Sz.-Nagy-Foiaş model theory for contraction operators [11] can be viewed as statements about representations of the disc algebra $\mathrm{Rat}(\bar{D})$ (D the unit disk). Thus the theory begun by Abrahamse and Douglas can be viewed as a generalization of the Sz.-Nagy-Foiaş theory to multiply-connected domains.

In this paper we shall deal with some of the specific questions concerning such representations raised by Abrahamse and Douglas in their paper. A representation σ is said to be of class C_{00} if σ is continuous from the topology of bounded pointwise convergence on R in $\mathrm{Rat}(\bar{R})$ to the double strong operator topology in $\mathscr{L}(\mathscr{H})$. A representation is said to be of class C_0 if its unique extension to $H^\infty(R)$ has a nontrivial kernel. It can be shown that these definitions are consistent with those of Sz.-Nagy and Foiaş for the case that $R = D$. In Section 2 of this paper we show that if $\sigma: \mathrm{Rat}(\bar{R}) \to \mathscr{L}(\mathscr{H})$ is a representation of class C_{00} such that $\sigma(z) = N + K$, where N is normal with spectrum contained in the boundary of R and K is trace class, then σ is of class C_0. This answers Question 6 of [4].

Associated with any completely contractive unital representation of $\mathrm{Rat}(\bar{R})$ (see the definition in Section 3) is a functional model analogous to the Sz.-Nagy-Foiaş functional model for a representation of the disc algebra. As in the disc case, the simplest form of the model occurs when the representation is C_{00}. The model is determined by a characteristic function, which in the disc case is uniquely determined by the representation. In the general case, as was pointed out by

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