REMARKS ON ABSOLUTELY SUMMING TRANSLATION INVARIANT OPERATORS FROM THE DISC ALGEBRA AND ITS DUAL INTO A HILBERT SPACE

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In this note among other results we prove the following

THEOREM 1. *Let* $f_j \in L^1$ *for* $j = 1, 2, \ldots$ *. Assume that*

$$(1) \qquad \sum_{j=1}^{\infty} \left| \int_0^{2\pi} f_j(t)\, h(t)\, dt \right| < +\infty \qquad \textit{for every } h \in H^{\infty}.$$

Then for every scalar sequence (m_k) *with* $\sum_{k=0}^{\infty} |m_k|^2 < +\infty$,

$$(2) \qquad \sum_{j=1}^{\infty} \sqrt{\sum_{k=0}^{\infty} |m_k \hat{f}_j(-k)|^2} < +\infty,$$

where $\hat{f}(k) = \frac{1}{2\pi} \int_0^{2\pi} f(t)\, e^{-ikt}\, dt$ *for* $k = 0, \pm 1, \pm 2, \ldots$.

By L^p $(0 < p \le \infty)$ we denote the space of equivalence classes of p-absolutely integrable with respect to the Lebesgue measure complex-valued measurable functions on $[0, 2\pi]$, and by $C_{2\pi}$ the space of 2π-periodic continuous complex-valued functions on $[0, 2\pi]$. For $f \in L^p$ we put $\|f\|_p = \left(\frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p\, dt\right)^{1/p}$ for $p \ge 1$ and $\|f\|_p = \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p\, dt$ for $0 < p < 1$. The Hardy spaces H^p $(1 \le p \le \infty)$ and the Disc Algebra A are defined by

$$H^p = \{f \in L^p : \hat{f}(k) = 0 \text{ for } k < 0\}, \qquad A = \{f \in C_{2\pi} : \hat{f}(k) = 0 \text{ for } k < 0\}.$$

In the language of absolutely summing operators Theorem 1 means that the adjoint of every translation invariant operator from H^2 into A is 1-absolutely summing. It is an open question whether every bounded linear operator from H^2 into A has 1-absolutely summing adjoint.

Our proof of Theorem 1 is indirect. Our argument uses the duality between nuclear and bounded operators and Theorem 2 below which asserts that a translation invariant operator $M : A \to H^2$ is nuclear if and only if it is p-absolutely summing for some p with $1 > p > 0$.

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