CRITICAL POINTS AND POINT DERIVATIONS ON M(G)

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Throughout this paper, let G be an arbitrary *nondiscrete* LCA group, and M(G) the convolution measure algebra of G (*cf.* [8] and [10]). We denote by $\Delta = \Delta_{M(G)}$ the maximal ideal space of M(G). Notice that Δ has a natural semigroup structure; in fact, if S denotes the structure semigroup of M(G), then Δ may be identified with $\hat{S}$, the semigroup of all continuous semicharacters of S [14].

In the present paper we shall study the existence of nontrivial continuous point derivations at certain elements of Δ. Recall that a point derivation at a given element $f \in \Delta$ is a linear functional D on M(G) such that

$$D(\mu * \nu) = (D\mu) \cdot \hat{\nu}(f) + (D\nu) \cdot \hat{\mu}(f), \qquad \mu, \nu \in M(G).$$

We shall say that such a D is continuous if it is continuous in the spectral radius norm of M(G). As is well-known, the existence of a nontrivial continuous point derivation at f implies that f is not a strong boundary point for the uniform closure of $M(G)^{\wedge}$ in $C(\Delta)$ (see [2; Chapter II, Exercise 12(e)]). On the other hand, the strong boundary points $f \in \Delta$ satisfy $|f|^2 = |f|$ and the Shilov boundary of M(G) is contained in the closure of all such f's ([14; p. 91]). Moreover, if $f \in \Delta$ and $|f|^2 \neq |f|$, then there exists a nontrivial continuous point derivation at f. In fact, letting $f = f_0|f|$ denote the polar decomposition of such an f ([14; p. 28]), we have that $z \to f_0|f|^z$ (Re $z > 0$) is an analytic map having the value f at $z = 1$; hence

$$\mu \to \frac{d}{dz}(\hat{\mu}(f_0|f|^z)) \Big|_{z=1}$$

is such a point derivation at f. We may therefore restrict our attention to those elements of Δ which have idempotent modulus. G. Brown and W. Moran [1] have recently proved that there exists a nontrivial continuous point derivation at the critical point of Δ which corresponds to the discrete topology of G. (For a generalization of this result, see [4].) In the present paper we shall prove as a consequence of our main result that the last result holds for every element of Δ whose modulus is a critical point different from the identity $1 \in \Delta$.

Now we introduce some notation. Given a Borel set E in G, let I(E) be the set of those measures μ in M(G) which satisfy $|\mu|(E + x) = 0$ for all $x \in G$, and let $R(E) = I(E)^{\perp}$ be the set of those measures in M(G) which are singular with respect to all members of I(E). Thus I(E) and R(E) are an L-ideal and an L-subspace of M(G), respectively, and M(G) can be decomposed into the direct sum of I(E)

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