INTERPOLATING SEQUENCES FOR HARDY AND BERGMAN CLASSES IN POLYDISKS

Eric P. Kronstadt and Charles W. Neville

1. INTRODUCTION AND STATEMENT OF RESULTS

Let U^n denote the unit polydisk in n dimensional complex space, $\mathbb{C}^n$. For $(E, |\cdot|)$ a non-trivial complex Banach space, $1 \leq p < \infty$, and $\alpha \geq 0$, we define the following Banach spaces: $\ell^\infty(E)$ is the space of bounded, E-valued sequences; $\ell^p(E)$ is the space of E-valued sequences satisfying $(\|(e_i)_{i=1}^\infty\|_p)^p \equiv \sum |e_i|^p < \infty$; $H^\infty(U^n, E)$ is the space of bounded analytic E-valued functions on U^n; $H^p(U^n, E)$ is the space of analytic E-valued functions on U^n satisfying

$$(\|f\|_p)^p \equiv \sup_{r<1} (2\pi)^{-n} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} |f(re^{i\theta_1}, \cdots, re^{i\theta_n})|^p d\theta_1 \cdots d\theta_n < \infty;$$

and $A^{p,\alpha}(U^n, E)$ is the space of analytic E-valued functions satisfying

$$(\|f\|_{A^{p,\alpha}})^p = ((\alpha+1)/\pi)^n \int_{U^n} |f(z)|^p \prod_{k=1}^{n} (1 - |z_k|^2)^\alpha d\nu(z) < \infty,$$

where $z = (z_1, z_2, \cdots, z_n)$, and $d\nu(z)$ is Lebesgue measure on U^n. When $E = \mathbb{C}$, these are the familiar sequence, Hardy, and Bergman spaces. (*Cf.* [6], [10], and [11].)

If $a = (a_1, \cdots, a_n) \in U^n$, and f is a function on U^n, define $T_a^\infty f = f(a)$; $T_a^p f = \left(\prod_{k=1}^n (1 - |a_k|^2)\right)^{1/p} f(a)$; and $T_a^{p,\alpha} f = \left(\prod_{k=1}^n (1 - |a_k|^2)\right)^{(\alpha+2)/p} f(a)$. The operators, T_a^∞, T_a^p, and $T_a^{p,\alpha}$ are the normalized point evaluation operators on $H^\infty(U^n, E)$, $H^p(U^n, E)$, and $A^{p,\alpha}(U^n, E)$ respectively. If

$$\mathscr{A} = (a_i)_{i=1}^\infty = ((a_{i1}, \cdots, a_{in}))_{i=1}^\infty$$

is a sequence in U^n define $T_{\mathscr{A}}^p f = (T_{a_i}^p f)_{i=1}^\infty$, for $1 \leq p \leq \infty$; and

$$T_{\mathscr{A}}^{p,\alpha} f = (T_{a_i}^{p,\alpha} f)_{i=1}^\infty, \quad \text{for } 1 \leq p < \infty \text{ and } \alpha \geq 0.$$

The fundamental questions of this paper are: When is $T_{\mathscr{A}}^p(H^p(U^n, E)) = \ell^p(E)$? When is $T_{\mathscr{A}}^{p,\alpha}(A^p(U^n, E)) = \ell^p(E)$?

The sequence $\mathscr{A}$ is said to be $H^p(U^n, E)$ or $A^{p,\alpha}(U^n, E)$ interpolating if $T_{\mathscr{A}}^p(H^p(U^n, E)) \supseteq \ell^p(E)$ or $T_{\mathscr{A}}^{p,\alpha}(A^{p,\alpha}(U^n, E)) \supseteq \ell^p(E)$. We remark, first, that if a

Received April 4, 1977.
The work of the first author was partially supported by N.S.F. Grant MPS 75 07940.

Michigan Math. J. 24 (1977).