DISCRETE CARLESON MEASURES AND SOME INTERPOLATION PROBLEMS

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1. INTRODUCTION

The study of sequences of distinct points in the open unit disc

$$\Delta = \{z\colon z \in \mathbb{C},\ |z| < 1\}$$

satisfying

$$(1) \qquad \sum |f(z_n)|(1 - |z_n|) \leq M\|f\| \quad \text{for all } f \in H^1;$$

or equivalently [4, p. 152],

$$(2) \qquad \sum |f(z_n)|^2(1 - |z_n|^2) \leq M\|f\|^2 \quad \text{for all } f \in H^2,$$

has been important in the study of interpolation problems in H^∞, the algebra of bounded analytic functions on Δ. In [6], Newman proved that if the sequence $\{z_n\}$ satisfies (1) and is *uniformly separated; i.e.*,

$$(3) \qquad \prod_{i \neq j} \left|\frac{z_i - z_j}{1 - \bar{z}_i z_j}\right| \geq \delta > 0, \qquad j = 1, 2, \cdots,$$

then for each $\{w_n\} \in \ell^\infty$, there exists $f \in H^\infty$ such that $f(z_n) = w_n$; that is, the sequence $\{z_n\}$ is a *universal interpolating sequence*. In [1], Carleson showed that the sequence $\{z_n\}$ is uniformly separated if and only if it is a universal interpolating sequence. A key step in Carleson's theorem was the proof that if $\{z_n\}$ is uniformly separated, then (1) is satisfied. Similarly, in another proof of Carleson's theorem by Shapiro and Shields [7], a key step was in establishing that if $\{z_n\}$ is uniformly separated, then the equivalent inequality (2) is satisfied.

In Section 2 we give a necessary and sufficient arithmetic condition on the sequence $\{z_n\}$ for (2) to be satisfied. As a corollary, we deduce a partial converse to the results of Carleson and of Shapiro and Shields. We show that if (2) holds, then the sequence $\{z_n\}$ is a finite union of uniformly separated subsequences.

The idea behind the inequalities (1) and (2) has been generalized in at least two different ways. In [2], Carleson considered general measures μ in the open unit disc and gave a necessary and sufficient geometric condition that

$$(4) \qquad \int |f(z)|^p d\mu \leq M\|f\|^p \quad \text{for all } f \in H^p.$$

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