JORDAN C*-ALGEBRAS

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INTRODUCTION

In his final lecture to the 1976 St. Andrews Colloquium of the Edinburg Mathematical Society, Professor Kaplansky introduced the concept of a Jordan C*-algebra (see below for definitions), pointed out its potential importance, and made the following conjecture. Let $\mathscr{A}_1$, $\mathscr{A}_2$ be unital Jordan C*-algebras and let $\phi: \mathscr{A}_1 \to \mathscr{A}_2$ be a surjective isometry with $\phi 1 = 1$; then ϕ is a Jordan *-isomorphism. In verifying this conjecture [15], extensive use was made of the deep results of Alfsen, Schultz, and Störmer [2] on JB-algebras.

It is easy to see that the self-adjoint part of a Jordan C*-algebra is a JB-algebra. The main part of this paper, Section 2, is devoted to establishing a converse result. *Each* JB-*algebra is the self-adjoint part of a unique Jordan* C*-*algebra.* First we establish the result for finite-dimensional algebras. This is not entirely straightforward and seems to require quite delicate arguments. Once this is accomplished; in particular, when we know of the existence of an exceptional Jordan C*-algebra, $\mathscr{M}_3^8$, whose self-adjoint part is M_3^8 (the exceptional Jordan algebra discovered by von Neumann, Jordan, and Wigner [6]), then the general result can be obtained quite quickly.

In the final section we consider ideals and quotients of Jordan C*-algebras and, applying the results of Section 2 and the main theorem of [2], show that for each Jordan C*-algebra $\mathscr{A}$ there exists a unique *-ideal $\mathscr{J}$ such that (i) $\mathscr{A}/\mathscr{J}$ can be isometrically *-isomorphically embedded into the special Jordan *-algebra of bounded operators on a complex Hilbert space and (ii) each 'factorial' representation of $\mathscr{A}$ which does not annihilate $\mathscr{J}$ is onto $\mathscr{M}_3^8$.

I would like to draw the attention of the reader to an interesting recent paper by Bonsall [3] in which he obtains a generalization of the Vidav-Palmer Theorem to special Jordan *-algebras.

1. BASIC PROPERTIES OF JORDAN C*-ALGEBRAS

Definition (Kaplansky). Let $\mathscr{A}$ be a complex Banach space and a complex Jordan algebra equipped with an involution *. Then $\mathscr{A}$ is a *Jordan* C*-*algebra* if the following four conditions are satisfied.

(i) $\|x \circ y\| \leq \|x\| \, \|y\|$ for all x and y in $\mathscr{A}$.

(ii) $\|z\| = \|z^*\|$ for all z in $\mathscr{A}$.

(iii) $\|\{zz^*z\}\| = \|z\|^3$ for all z in $\mathscr{A}$.

(Here $\{abc\}$ is the Jordan triple product as defined on page 36 [5].)

(iv) Each norm-closed, associative *-subalgebra of $\mathscr{A}$ is a C*-algebra

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