EXTRINSIC SPHERES IN COMPACT SYMMETRIC SPACES ARE INTRINSIC SPHERES

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1. INTRODUCTION

An n-dimensional submanifold, $n \geq 2$, of an arbitrary Riemannian manifold is called an *extrinsic sphere* if it is umbilical and has nonzero parallel mean curvature vector. An n-dimensional Riemannian manifold is called an *intrinsic sphere* if it is locally isometric to a standard sphere in Euclidean space. Since extrinsic spheres are natural analogues of ordinary spheres in Euclidean spaces from the extrinsic point of view, it is natural to ask when an extrinsic sphere is an intrinsic sphere. In [2], we have proved that a complete, simply connected, extrinsic 2n-sphere in any Kähler manifold is an intrinsic sphere if its normal connection is flat. The main purpose of this paper is to prove the following.

THEOREM 1. *An extrinsic sphere in a compact symmetric space is an intrinsic sphere.*

THEOREM 2. *Let* M *be an extrinsic sphere in a compact symmetric space* $\tilde{M}$. *Then* $\dim M \leq d_{\tilde{M}}$, *where* $d_{\tilde{M}}$ *is the maximal dimension of all totally geodesic submanifolds of constant sectional curvature in* $\tilde{M}$.

Theorem 2 improves the results in [3] for the compact case.

2. PRELIMINARIES

Let M be an n-dimensional submanifold of a symmetric space $\tilde{M}$ with metric g, and let ∇ and $\tilde{\nabla}$ be the covariant differentiations on M and $\tilde{M}$, respectively. Then the second fundamental form σ is defined by $\sigma(X, Y) = \tilde{\nabla}_X Y - \nabla_X Y$, where X and Y are vector fields tangent to M and σ is a normal-bundle-valued symmetric 2-form on M. For a vector field ξ normal to M, we write $\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi$, where $-A_\xi X$ (respectively, $D_X \xi$) denotes the tangential component (respectively, the normal component) of $\tilde{\nabla}_X \xi$. A normal vector field ξ is said to be *parallel* if $D\xi = 0$. The submanifold is said to be *umbilical* if $\sigma(X, Y) = g(X, Y) H$, where $H = (\text{trace}\ \sigma)/n$ is the *mean curvature vector* of M in $\tilde{M}$. If $\sigma = 0$, M is said to be *totally geodesic* in $\tilde{M}$.

Let R^N, R, and $\tilde{R}$ be the curvature tensors associated with D, ∇, and $\tilde{\nabla}$, respectively. For example, $R(X, Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$. The submanifold M is locally symmetric if $\nabla R = 0$, and the normal connection of M in $\tilde{M}$ is flat if $R^N = 0$.

For the second fundamental form σ, we define the covariant derivative, denoted by $\bar{\nabla}_X \sigma$, to be

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