COBORDISM CLASSES REPRESENTED BY FIBERINGS WITH FIBER $\mathbb{R}P(2k+1)$

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1. INTRODUCTION

Let k be a nonnegative integer. Let $\eta_{n-k}(BO(k+1))$ be the unoriented cobordism group of real $(k+1)$-plane bundles over closed smooth $(n-k)$-dimensional manifolds. Let $\sigma_n^k\colon \eta_{n-k}(BO(k+1)) \to \eta_n$ be the homomorphism defined by assigning to the $(k+1)$-plane bundle ξ over M^{n-k} the cobordism class of the total space $\mathbb{R}P(\xi)$ of the associated projective space bundle. Many problems in cobordism theory can be reduced or related to the computation of this homomorphism. For instance, Stong [6; 8.4] proved that the image of σ_n^k is the set of cobordism classes in η_n which are represented by the total space of a fibering $\mathbb{R}P(k) \xrightarrow{i} M^n \xrightarrow{\pi} B^{n-k}$ which is totally nonhomologous to zero. Another example of the usefulness of σ_n^k was described in [1]: Let J_n^k be the set of cobordism classes in η_n which are represented by a manifold admitting an involution whose fixed point set is $(n-k)$-dimensional. Then the image of σ_n^k contains J_n^k, which in turn contains the image of σ_n^{2k-1}.

The main results of this paper are the following:

PROPOSITION 2.3. *The image of* σ_n^3 *equals the set of classes in* η_n *which are represented by a fibering with fiber* $\mathbb{R}P(3)$, *and is the set of classes* α *in* η_n *with* $w_1^j w_{n-j}(\alpha) = 0$ *for all* j, $0 \le j \le n$.

PROPOSITION 4.4. *The image of* σ_n^5 *equals* J_n^3, *and is the set of classes* α *in* η_n *with* $w_1^j w_{n-j}(\alpha) = w_1^{i-5} w_{n-i} s_5(\alpha) = 0$ *for all* j *and* i, $0 \le j \le n$, $5 \le i \le n$.

2. THE IMAGE OF σ_n^3

PROPOSITION 2.0. *Let* $f\colon M^n \to B^b$ *be a smooth map and let* $F = f^{-1}(p)$ *be the inverse image of a regular value of* f. *Let* $i\colon F \to M$ *be the inclusion. Then* $i_*[F] = f^*[B] \cap [M]$.

Proof. By examining tubular neighborhoods of F and p, we see by naturality that $f^*[B]$ is equal to what Milnor and Stasheff call the dual cohomology class to F in M [4, page 120]. The proposition then follows from [4; Problem 11-c].

COROLLARY. *If* $F^f \xrightarrow{i} M \xrightarrow{\pi} B$ *is a smooth fibering, then for any class* $x \in H^f(M; \mathbb{Z}_2)$ *the numbers* $\langle i^*(x), [F]\rangle$ *and* $\langle x \cup \pi^*[B], [M]\rangle$ *are equal.*

Received October 8, 1976. Revision received April 27, 1977.
This research was supported in part by the National Science Foundation, Grant MCS 76-06373.

Michigan Math. J. 24 (1977).