EXTRINSIC SPHERES IN IRREDUCIBLE HERMITIAN SYMMETRIC SPACES

Chorng-Shi Houh

1. INTRODUCTION

Let M be an irreducible Hermitian symmetric space. Then $\tilde{M}$ is simply connected and its canonical Hermitian structure is Kählerian. Let $2m$ be the real dimension of $\tilde{M}$. An n-dimensional submanifold N of $\tilde{M}$ is called an extrinsic sphere if it is umbilical and has parallel, nonzero mean curvature vector. In Remark 2 of [2], Chen indicated that if the rank of $\tilde{M}$ is ℓ, then $\tilde{M}$ admits extrinsic spheres of dimensions $\leq \ell - 1$ with flat normal connections; namely, extrinsic spheres of maximal flat totally geodesic submanifolds of $\tilde{M}$. In this paper we investigate extrinsic spheres with flat normal connections in irreducible Hermitian symmetric spaces and shall prove the following.

THEOREM. *If* N *is an* n*-dimensional* $(n \geq 2)$ *complete, simply connected extrinsic sphere with flat normal connection in an irreducible Hermitian symmetric space* $\tilde{M}$, *then* $n \leq \operatorname{rank} \tilde{M}$ *and* N *is isometric to a standard* n*-sphere.*

2. PRELIMINARIES

$\tilde{M}$ is always assumed to be an irreducible Hermitian symmetric space of real dimension $2m$ $(m > 1)$. Let J and g be the complex structure and Kähler metric of $\tilde{M}$, let N be an n-dimensional submanifold of $\tilde{M}$, and let $\tilde{\nabla}$ and ∇ be the covariant differentiations on $\tilde{M}$ and N, respectively. The second fundamental form h of N in $\tilde{M}$ is defined by $h(X, Y) = \tilde{\nabla}_X Y - \nabla_X Y$, where X, Y are vector fields tangent to N. Then h is symmetric, with values in the normal bundle. For a vector field ξ normal to N we write $\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi$, where $-A_\xi X$ and $D_X \xi$ denote the tangential and normal components of $\tilde{\nabla}_X \xi$. If $D_X \xi = 0$, ξ is said to be *parallel*. If $h(X, Y) = g(X, Y)H$, where $H = (\operatorname{trace} h)/n$ is the mean curvature vector of N in $\tilde{M}$, N is said to be *umbilical*. Let $\tilde{R}$, R, and $R^\perp$ be the curvature tensors associated with $\tilde{\nabla}$, ∇, and D, respectively. Let

$$(\overline{\nabla}_X h)(Y, Z) = D_X(h(Y, Z)) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z)$$

for X, Y, Z tangent to N. Then the equations of Codazzi and Ricci are

$$(\tilde{R}(X, Y)Z)^\perp = (\overline{\nabla}_X h)(Y, Z) - (\overline{\nabla}_Y h)(X, Z);$$

$$\tilde{R}(X, Y; \xi, \eta) = R^\perp(X, Y, \xi, \eta) - g([A_\xi, A_\eta]X, Y),$$

Received March 23, 1977. Revision received May 16, 1977.

Michigan Math. J. 24 (1977).