A SELECTOR PRINCIPLE FOR Σ_1^1 EQUIVALENCE RELATIONS

John P. Burgess

Let $J = {}^{\omega}2$, the space of functions from the set ω of natural numbers to $2 = \{0, 1\}$ with its topology as a countable product of two-point discrete spaces. We assume familiarity with the hierarchy of Σ_n^1, Π_n^1 and Δ_n^1 subsets of J and its finite Cartesian powers J^k. (See *e.g.*, [13], Chapters 14-16.) We will be interested in equivalence relations E on J which are Σ_1^1 as subsets of J^2. A *selector set* or *transversal* for an equivalence E on J is a subset $S \subseteq J$ containing exactly one element of each E-equivalence class. Our goal is to determine the set-theoretic strength of the following Selector Principle:

(*) Every Σ_1^1 equivalence relation on J has a Δ_2^1 selector.

(Let us note right away that any Σ_2^1 selector S for a Σ_1^1 (or even Σ_2^1) equivalence E is automatically Π_2^1 and hence Δ_2^1, since

$$S = \{x: \neg\exists y(y \in S \ \&\ xEy \ \&\ x \neq y)\}.$$

Thus in (*) we could have written Σ_2^1 for Δ_2^1 without affecting the strength of the principle.)

It has long been known that (*) is consistent with, but independent of, the usual axioms (ZFC) of set theory. Work of D. Myers [12] provides more detailed information. The main contribution of the present paper is as follows: *It is well known that if every real is constructible, then every Σ_1^1 equivalence relation on the reals has a Σ_2^1 selector; the converse is not provable in* ZFC. This result was announced in [2].

In our work we make use of the following Ramsey-style theorem of Galvin: *Let the set* $[J]^2$ *of two-element subsets of* J *be partitioned into finitely many pieces in a nice enough way* (so that for each piece A the corresponding subset

$$\{(x, y): \{x, y\} \in A\}$$

of J^2 has the Baire property). *Then there is a perfect subset* P *of* J *such that all two-element subsets of* P *belong to the same piece of the partition.* Galvin's result was announced in [3] and [4]. Overlooking his work, we rediscovered it and announced it in [2]. Since no proof has so far been published, with Prof. Galvin's kind permission we are including one here.

Section 1 of the present paper contains a survey of known results concerning the status of (*). Section 2 contains a proof of the partition theorem mentioned above, and Section 3 a proof of Con(ZFC + (*) + $\neg$(Every real is constructible)).

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