A CHARACTERIZATION OF NON-FIBERED KNOTS

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INTRODUCTION

A tame knot k in S^3 is *fibered* if its complement fibers over S^1. By the work of Neuwirth [5] and Stallings [7], an equivalent condition is that the commutator subgroup of $\pi_1(S^3 - k)$ be finitely generated. In this paper, we show that a certain subgroup of a knot group is its own normalizer if and only if the corresponding knot is non-fibered. To be precise, our main theorem may be stated as follows:

THEOREM. *Let* k *be a tame non-fibered knot in* S^3, *and let* F *be a minimal spanning surface* [4, Section 7] *of* k. *Let* i: $(S^3 - F) \to (S^3 - k)$ *be the inclusion map, and set* $U = i_*(\pi_1(S^3 - F)) \subseteq \pi_1(S^3 - k) = G$. *Then* U *is its own normalizer in* G.

We remark that when k is a fibered knot, the subgroup U is just G', the commutator subgroup of G, which is normal in G; in particular, since G' is proper, U is not equal to its own normalizer in this case. We also note that in any case $U \subseteq G'$. Hence, our theorem implies that when k is non-fibered, $\text{Norm}(U) = U \subsetneq G'$.

After proving our main theorem, we will use it to show that certain knots have infinitely many non-isotopic minimal spanning surfaces. More precisely, we construct, for any composite knot $K = k_1 \# k_2$, an infinite family of minimal spanning surfaces, and then, by applying our theorem to the knots k_1 and k_2, we show that if k_1 and k_2 are non-fibered, no two of the minimal spanning surfaces are ambient isotopic by an isotopy which leaves K fixed at each level.

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PROOF OF THE THEOREM

Split S^3 along F to obtain a manifold whose boundary consists of two copies of F, say F_1 and F_2. The inclusions of F_1 and F_2 into this manifold induce homomorphisms $f_1: \pi_1(F) \to \pi_1(S^3 - F)$ and $f_2: \pi_1(F) \to \pi_1(S^3 - F)$. Since F is minimal, both f_1 and f_2 are injective, by Dehn's lemma and the loop theorem [5, p. 28]. If either f_1 or f_2 were surjective, then, by the Brown product theorem [1] (see also [7, Sections 6-10]), $(S^3 - F) = (\text{int } F) \times [0, 1]$, so that k would be a fibered knot. Therefore, since k is non-fibered, neither f_1 nor f_2 is surjective.

If we set $G = \pi_1(S^3 - k)$, $H = \pi_1(S^3 - F)$, and $A = f_1(\pi_1(F))$, and if we let ϕ be the isomorphism $f_2 \circ f_1^{-1}$ between $A = f_1(\pi_1(F))$ and $\phi(A) = f_2(\pi_1(F))$, then Van Kampen's theorem implies that G is the HNN group

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