GOLDIE CENTRALIZERS OF SEPARABLE SUBALGEBRAS

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1. INTRODUCTION

In this paper we shall discuss some relations between the structure of an algebra R and a certain subalgebra of R.

Before stating our results, let us recall some of the definitions and known results. Let R be a ring. The center of R is denoted Z(R). The ring R is *semiprime* if R has no nontrivial nilpotent ideals; it is *prime* if the relation AB = 0 for ideals A and B implies A = 0 or B = 0. The ring R is a (right) *Goldie* ring provided R satisfies the maximum condition on right annihilators and R contains no infinite direct sum of right ideals.

For x in R, we set

$$r(x) = \{y \in R \mid xy = 0\} \qquad \text{and} \qquad \ell(x) = \{y \in R \mid yx = 0\}.$$

An element x of R is *regular* if $r(x) = \ell(x) = 0$.

If A is a subset of R, the *centralizer* of A in R is

$$C_R(A) = \{x \in R \mid xa = ax \text{ for all } a \in A\}.$$

In [8], S. Montgomery explored some relations between the structure of an algebra R over a field and the structure of the centralizer $C_R(A)$ of a finite-dimensional separable subalgebra A. In [2] we showed that if R is a semiprime n-torsion-free ring, a is an element of R such that $a^n \in Z(R)$, and $C_R(A)$ is a semiprime (prime) Goldie ring, then R is a semiprime (prime) Goldie ring.

In this paper we shall show that if R is a semiprime algebra over a field and A is a finite-dimensional separable subalgebra, then $C_R(A)$ is a Goldie ring if and only if R is a Goldie ring.

As a consequence we can extend the results of [2] to the case where Z(R) is a field and a is a zero of a separable polynomial over Z(R).

We shall use localizations later. Let T be a nonempty set of regular elements of R. Then T is a *right denominator set* if T is closed under multiplication and if $xT \cap tR \neq 0$ for each nonzero x in R and each t in T. P. M. Cohn [3, p. 21], has shown that the *localization* of R by T,

$$R_T = \{xt^{-1} \mid x \in R, t \in T\}$$

exists for such a T and $R \subset R_T$.

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