SMOOTH S^1-MANIFOLDS IN THE HOMOTOPY TYPE OF $\mathbb{C}P^3$

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0. INTRODUCTION

Four years ago T. Petrie [6] conjectured that *if* X *is a closed, smooth,* 2n-*dimensional homotopy* $\mathbb{C}P^n$ *that admits a nontrivial action of* S^1, *and if* $h: X \to \mathbb{C}P^n$ *is a homotopy equivalence, then* h *preserves Pontrjagin classes.*

In the present paper we prove the conjecture for the case $n = 3$:

THEOREM 0.1. *Let* X *be a closed, smooth* S^1*-manifold such that* $X^{S^1} \neq X$, *and let* $f: X \to \mathbb{C}P^3$ *be a homotopy equivalence. Then*

$$f^* \hat{\mathscr{A}}(\mathbb{C}P^3) = \hat{\mathscr{A}}(|X|),$$

where $|X|$ *denotes the underlying smooth manifold of* X,

$$\hat{\mathscr{A}}(|X|) = (x_i/2)(\sinh x_i/2)^{-1} \in H^*(|X|; \mathbb{Q}),$$

and the elementary symmetric functions of the x_i^2 *give the Pontrjagin classes of* $|X|$. *In particular,* f *preserves the Pontrjagin classes of* $|X|$.

Furthermore, a theorem of D. Montgomery and C. T. Yang [5] implies that there is a bijective application

$$P: \mathbb{Z} \to \{\text{diffeomorphism classes of smooth manifolds homotopy equivalent to } \mathbb{C}P^3\}$$

such that, for every $\alpha \in \mathbb{Z}$,

$$p_1(P(\alpha)) = (24\alpha + 4)z^2,$$

where p_1 is the first Pontrjagin class and z is a generator of $H^2(\mathbb{C}P^3)$.

THEOREM 0.2. *A closed smooth* S^1 *manifold* X, *homotopy-equivalent to* $\mathbb{C}P^3$ *and such that* $X^{S^1} \neq X$, *is diffeomorphic to* $\mathbb{C}P^3$.

Theorem 0.1 follows from Theorem 2.1, as indicated subsequently. This is intimately related to the proof of Theorem 1.3, which completely determines the rational torsion-free equivariant K-theory of X.

1. EQUIVARIANT COHOMOLOGIES

Let G be a compact abelian Lie group that is topologically cyclic, in other words, such that there exists a dense generator g in G. Let R(G) be the representation ring of G. Let Z be a closed, smooth G-manifold such that $Z^G \neq Z$, and let $\hat{K}_G^*(Z)$ be the quotient of the equivariant K-theory $K_G^*(Z)$ by its R(G)-torsion.

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