REGULAR NEIGHBORHOODS OF ORIENTABLE 3-MANIFOLDS

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1. INTRODUCTION

If M^m and Q^q are PL manifolds with $M \subseteq Q$, any two regular neighborhoods of M in Q are isotopic relative to M [1]. The matter of classifying different regular neighborhoods of a fixed M has been studied by C. P. Rourke and B. J. Sanderson [6], who construct a universal classifying space $BPL_{\tilde{q}}$; different neighborhoods correspond to homotopy classes of (Δ-) maps of M into $BPL_{\tilde{q}}$.

In this paper, the different regular neighborhoods of orientable 2- and 3-manifolds will be constructed and compared. As is usually the case, two regular neighborhoods N_1 and N_2 of a manifold M will be considered the same ($N_1 \cong N_2$) if and only if there exists a PL homeomorphism $h: N_1 \to N_2$ such that $h(x) = x$ for all $x \in M$. It will be seen in these two cases that the distinct orientable regular neighborhoods are in one-to-one correspondence with the elements of $H^2(M; \mathbb{Z}_2)$. A similar classification exists for tubular neighborhoods of differentiably embedded closed orientable 2- and 3-manifolds; the techniques are easily adapted to the differentiable case.

The notation and definitions used here will be consistent with those found in J. F. P. Hudson's book [4]. The boundary of a manifold M will be denoted by ∂M, and Δ^n will be the standard n-simplex; further, we write

$$I = [0, 1], \qquad I^1 = [-1, 1], \qquad I^n = I^{n-1} \times I^1, \qquad S^n = \partial I^{n+1}.$$

If L and K are simplicial complexes with $L < K$, $cx(K - L)$ will be used to denote the smallest subcomplex of K that contains $K - L$; by K'' (rel L) we shall mean a second derived subdivision of K relative to L. All maps and manifolds will be PL. In particular, if V and V' are PL manifolds, a *concordance* is a PL homeomorphism $H: V \times I \to V' \times I$ that maps $V \times \{i\}$ homeomorphically to $V' \times \{i\}$ for $i = 0, 1$. Two homeomorphisms $f_0, f_1: V \to V'$ are said to be *concordant relative to* $X \subseteq V$ in case there exists a concordance $H: V \times I \to V' \times I$ with $H_0 = f_0$ and $H_1 = f_1$ such that $H(x, t) = (H_0(x), t)$ for all $x \in X$. In this event, H is said to be *fixed* on X.

Block bundles [6] are a key tool in the construction, as are Δ-sets and their homotopy groups [7]. Of particular importance are the Δ-sets $PL_{\tilde{q}}$, whose k-simplexes are block isomorphisms of $\Delta^k \times I^q$ onto itself, and $PL_q(I)$, the sub-Δ-set of fibre-preserving block isomorphisms. Each of these sets has two components; however, the symbols $PL_{\tilde{q}}$ and $PL_q(I)$ will be used here (incorrectly) to represent only the component containing the identity map.

If K is a simplicial complex (with the set of vertices totally ordered), $\underline{K}$ will represent the associated Δ-set. Each n-simplex $A \in K$ will be identified with Δ^n according to the order of its vertices by a map σ^n. This identification induces for

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