AN L_2 ANALYTIC FOURIER-FEYNMAN TRANSFORM

R. H. Cameron and D. A. Storvick

INTRODUCTION

The concept of an *analytic Fourier-Feynman transform* was introduced in 1972 by M. D. Brue [2], and it was defined essentially as in (0.2) below. It was based on the analytic Wiener and Feynman integrals [3], for which we now give simplified definitions sufficiently general for this paper.

Definition. Let C[a, b] be the space of real continuous functions x(·) on [a, b] for which x(a) = 0. Let F be a functional such that the Wiener integral

$$J(\lambda) = \int_{C[a,b]} F(\lambda^{-1/2} x)\, dx$$

exists for almost all real $\lambda > 0$. If there exists a function $J^*(\lambda)$ analytic in the half-plane $\Re\,\lambda > 0$ such that $J^*(\lambda) = J(\lambda)$ for almost all real $\lambda > 0$, then we define this "essential analytic extension" of J to be the *analytic Wiener integral of* F *over* C[a, b] *with parameter* λ, and for $\Re\,\lambda > 0$ we write

$$\int_{C[a,b]}^{\text{anw}_\lambda} F(x)\, dx = J^*(\lambda)\,.$$

Definition. Let q be a real parameter ($q \neq 0$), and let F be a functional whose analytic Wiener integral exists for $\Re\,\lambda > 0$. Then, if the following limit exists, we call it the *analytic Feynman integral of* F *over* C[a, b] *with parameter* q, and we write

$$\text{(0.1)} \qquad \int_{C[a,b]}^{\text{anf}_q} F(x)\, dx = \lim_{\substack{\lambda \to -iq \\ \Re\,\lambda > 0}} \int_{C[a,b]}^{\text{anw}_\lambda} F(x)\, dx\,.$$

On the basis of these definitions, we can define Brue's transform as follows:

Definition. If $q \neq 0$ and if for each $y \in C[a, b]$ the analytic Feynman integral

$$\text{(0.2)} \qquad T_q^* F \equiv \int_{C[a,b]}^{\text{anf}_q} F(x + y)\, dx$$

exists, then $T_q^* F$ is called the *analytic Fourier-Feynman transform of* F.

Actually, Brue used a slightly more general definition of the analytic Feynman integral, but restricted the definition of his transform to the case $q = -1$, using the case $q = 1$ as the inverse transform.

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