# EXISTENCE OF MARKOV PROCESSES ASSOCIATED WITH NONCONTRACTION SEMIGROUPS

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## 1. INTRODUCTION

The connection between contraction semigroups and Markov processes has been extensively studied. However, the relation between noncontraction semigroups and Markov processes remains virtually unexplored. The purpose of this article is to generalize the definition of Markov processes so that a large class of noncontraction semigroups is associated with Markov processes. Although the results of this article can be extended to the cases where the state spaces are more general, we shall consider only the case where the state space Z is the complex plane or a subset of it and is topologically equivalent to a complete separable metric space.

For a measure Q on a measurable space $(M, \mathscr{B})$, let $|Q|$, $Q^r$, and $Q^i$ denote the total variation, the real part, and the imaginary part of Q, respectively. If Q is a signed measure, let $Q^+$ and $Q^-$ denote the positive and negative parts of Q.

An *abstract probability space* $(M, \mathscr{B}, Q)$ is a finite measure space with $Q(M) = 1$. Q is called an *abstract probability measure*. If $\mathscr{G}$ is a sub-$\sigma$-field of $\mathscr{B}$, let $Q_{\mathscr{G}}$ denote the restriction of Q to $\mathscr{G}$. An element $G \in \mathscr{G}$ is called a *$\mathscr{G}$-null set* if $|Q_{\mathscr{G}}|(G) = 0$. Note that the previous definition of a $\mathscr{G}$-null set G requires also the condition $|Q|(G) \neq 0$ (see [5]). Here we drop this condition, because a set G with $|Q|$-measure 0 can be neglected. The set $N_{\mathscr{G}} = \{(dQ_{\mathscr{G}}/d|Q|_{\mathscr{G}}) = 0\}$ is a $\mathscr{G}$-null set that contains every other $\mathscr{G}$-null set $|Q|$-a.e. A $\mathscr{G}$-measurable r.v. (random variable) Y is said to be *$\mathscr{G}$-integrable* if $|Y|$ is $|Q_{\mathscr{G}}|$-integrable. The conditional expectation $E(X \mid \mathscr{G})$ (see [5]) of a r.v. X given a sub-$\sigma$-field $\mathscr{G}$ is the $|Q|$-a.e. uniquely defined r.v. Y, if it exists, that satisfies the conditions

(1.1) $$Y \text{ is } \mathscr{G}\text{-integrable},$$

(1.2) $$Y = 0 \text{ on } N_{\mathscr{G}},$$

(1.3) $$\int_G Y\,dQ = \int_G X\,dQ \quad \text{for each } G \in \mathscr{G}.$$

If X is bounded, then $E(X \mid \mathscr{G})$ exists if and only if the condition

(1.4) $$\int_G X\,dQ = 0$$

is satisfied for each $\mathscr{G}$-null set G (see [5, Theorem 2.1]). In the case where $|Q_{\mathscr{G}}|$ and $|Q|_{\mathscr{G}}$ are equivalent, condition (1.4) is always satisfied and hence $E(X \mid \mathscr{G})$ always exists. The following definition is a generalization of the classical definition of a Markov process.

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