LOCALLY COMPACT CONVERGENCE SPACES

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INTRODUCTION

Since local compactness plays a central role in topology and analysis, it is natural to investigate this concept in the more general realm of convergence spaces. Recent results (see for example Theorem 3.2 and 3.6 of [5]) indicate that local compactness will be of increasing importance in the study of convergence spaces.

In this paper, we show that the topological spaces known as k-spaces (see [1], [2], and [9]) are precisely the topological modifications of the locally compact convergence spaces. We establish, as a special case of a more general theorem on products of convergence spaces, that the product of a k-space with a locally compact is a k-space. In response to a question by A. Arhangel'skiĭ and S. P. Franklin [1], we construct a locally compact convergence space X_0 whose topological modification is a k-space of arbitrarily large ordinal index. We use the same example to extend the results of [8] by showing, among other things, that a function space $C_c(X)$ can have an arbitrarily long decomposition series when X is a locally convex topological linear space.

1. PRELIMINARIES

The reader is asked to refer to [6] for definitions, notation, and terminology pertaining to convergence spaces. As in [6], the term *space* will always mean "convergence space," and the term "ultrafilter" will be abbreviated to "u.f.". Unlike [6], the present paper will not make the assumption that all spaces are Hausdorff spaces.

Some additional definitions and terminology are needed for our present investigation. Let X be a space. If $A \subseteq X$ and σ is an ordinal number, then we denote by $\mathrm{cl}_X^{\sigma} A$ the σth iteration of the closure of A; this is defined to be $\mathrm{cl}_X\, \mathrm{cl}_X^{\sigma-1} A$ if $\sigma - 1$ exists, and $\bigcup \{\mathrm{cl}_X^{\rho} A : \rho < \sigma\}$ if σ is a limit ordinal. The smallest ordinal α such that $\mathrm{cl}_X^{\alpha+1} A = \mathrm{cl}_X^{\alpha} A$ for all $A \subseteq X$ is called the *length of the decomposition series of* X, and we denote it by $\ell_D(X)$.

A *pseudo-topological* space is a space with the property that $\mathfrak{F} \to x$ whenever each u.f. finer than $\mathfrak{F}$ converges to x. For any space X, let ρX be the space defined on the same underlying set as follows: $\mathfrak{F} \to x$ in ρX if and only if $\mathfrak{G} \to x$ in X for each u.f. $\mathfrak{G} \geq \mathfrak{F}$. The space ρX is the finest pseudo-topological space coarser than X, and it is called the *pseudo-topological modification* of X. Note that X and ρX have the same u.f. convergence.

A space is said to be *pretopological* if the X-neighborhood filter $\mathcal{V}_X(x)$ at x (obtained by intersecting all filters that converge to x) converges to x for all $x \in X$. Any set in $\mathcal{V}_X(x)$ is called an X-*neighborhood* of x. The *pretopological*

Received May 5, 1975.

Michigan Math. J. 22 (1975).