ERGODIC PROPERTIES OF EXPANSIVE AUTOMORPHISMS

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INTRODUCTION

In the study of expansive homeomorphisms, first made under the term "unstable homeomorphisms" (see [3], for example), many reports on expansive homeomorphisms and automorphisms have been published. However, in spite of a perhaps interesting topic, ergodic properties of expansive automorphisms are unknown, except in special cases. In this paper, we show that *expansive automorphisms of compact, connected, finite-dimensional abelian groups are* K-*automorphisms.*

1. PRELIMINARY LEMMAS

Throughout this paper, given an automorphism of an abelian group, we shall denote the restrictions on invariant subgroups and the automorphisms induced on factor groups by invariant subgroups by the symbol used for the original automorphism.

LEMMA A. *Let* G *be a countable, torsion-free, discrete, abelian group, and let* U *be an automorphism of* G. *Then there exist a minimal divisible extension* $\overline{G}$ *of* G *and an automorphism* $\overline{U}$ *of* $\overline{G}$ *such that* $\overline{U}$ *is an extension of* U. *Furthermore, if* U *has no finite orbit, then* $\overline{U}$ *has no finite orbit.*

Proof. Write $G = \{h_1, h_2, \cdots\}$. For each integer j and each positive integer n, let $[\hat{h}_{nj}]$ denote the free cyclic group generated by a new element $\hat{h}_{nj}$, and let $\langle U^j h_n \rangle$ denote the cyclic group generated by $U^j h_n$. Then we can construct a natural homomorphism ϕ_n from the direct-product group $W_n = \bigotimes_{j=-\infty}^{\infty} [\hat{h}_{nj}]$ onto the subgroup $\prod_{j=-\infty}^{\infty} \langle U^j h_n \rangle$ of G via the correspondence $\hat{h}_{nj} \to U^j h_n$. Let U_n' denote the automorphism of W_n defined by $\hat{h}_{nj} \to \hat{h}_{nj+1}$; then $\phi_n U_n' = U\phi_n$. Since

$$\prod_{n=1}^{\infty} \prod_{j=-\infty}^{\infty} \langle U^j h_n \rangle = G,$$

there is a homomorphism ϕ from the direct-product group $W = \bigotimes_1^\infty W_n$ onto G such that $\phi\{k_n\} = \{\phi_n k_n\}$ for each $\{k_n\} \in W$ and $\phi U' = U\phi$, where $U'\{k_n\} = \{U_n' k_n\}$. It is easy to see that U' is an automorphism of W. Hence (G, U) is a factor of (W, U'), and if we denote by K the kernel of ϕ, then (G, U) is isomorphic to (W/K, U').

Now there is a divisible extension $\overline{W} = \bigotimes_{n=1}^{\infty} \bigotimes_{j=-\infty}^{\infty} Q_{nj}$ of W, where Q_{nj} is an abelian group isomorphic to the additive group consisting of all rational numbers, and since W is torsion-free, there is an automorphism $\overline{U}'$ of $\overline{W}$ that is an extension of U'. Because $\overline{W}/K$ is a divisible extension of W/K, $\overline{U}'$ on $\overline{W}/K$ is an

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