EMBEDDINGS OF k-ORIENTABLE MANIFOLDS

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1. INTRODUCTION

Let M be a closed, k-connected, smooth, n-dimensional manifold, and let M_0 denote M minus a point $x_0 \in M$. In [2], J. C. Becker and H. Glover showed that for $j \leq 2k$ and $2j \leq n - 3$, the manifold M embeds in R^{2n-j} if and only if M_0 immerses in R^{2n-j-1}. We shall extend this result to $j = 2k + 1$ by placing an additional condition of orientability on M.

A vector bundle is called k-*orientable* if its restriction to the k-skeleton of its base is stably fibre-homotopy-trivial. A manifold is k-orientable if its tangent bundle is k-orientable.

Letting M be (k + 1)-orientable with $k \leq (n - 5)/4$, we state our main theorem.

THEOREM 1.1. M *embeds in* $R^{2n-2k-1}$ *if and only if* M_0 *immerses in* $R^{2n-2k-2}$.

This result reduces an embedding problem to one involving an immersion in which the top obstruction vanishes.

As applications we obtain the following.

THEOREM 1.2. *Let* M *be an* n-*dimensional, simply-connected spin manifold with* $n \equiv 3 \pmod 4$ *and* $n \geq 11$. *Then* M *embeds in* R^{2n-3}.

Proof. It is sufficient to show that the associated bundle with fibre $V_{m,m-n+4}$ has a cross-section, for large m. The obstructions to such a cross-section lie in $H^{i+1}(M_0; \pi_i(V_{m,m-n+4}))$. If $i < n - 4$, then $\pi_i = 0$. For $i = n - 4$, the obstruction $\overline{w}_{n-3}$ is 0, by [7]. The homotopy group π_{n-3} is 0, by [6]. By connectedness, $H^{n-1}(M_0) = 0$, and finally, $H^n(M_0) = 0$.

COROLLARY 1.3. *If* M *is a closed, almost parallelizable,* k-*connected* n-*manifold and* $k \leq (n - 5)/4$, *then* M *can be embedded in* $R^{2n-2k-1}$.

The corollary follows from the fact that M is (n - 1)-orientable and that by [4] M_0 can be immersed in R^n. This corollary extends a result of R. de Sapio [8], for some values of k.

2. ORIENTABILITY

Let $\mathscr{E}$ be a spectrum as defined in [10]. Let $\mathscr{S}$ denote the sphere spectrum, and let $\mathscr{S}^k$ denote the k-stem spectrum. (We obtain $(S^n)^k$ from S^n by killing the homotopy group for $i \geq n + k$ with the inclusion map $\lambda: S^n \to (S^n)^k$.) As in [10], we have a generalized homology and cohomology theory defined by

Received November 11, 1973.

Michigan Math. J. 21 (1974).