ALGEBRAICALLY SEPARABLE EXTENSIONS OF BANACH ALGEBRAS

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Let A be a commutative complex Banach algebra with identity, and let X(A) be its carrier space. In this paper we explore the relation between the finite-fibered covering spaces Y of X(A) and the faithful, commutative, separable algebras over the commutative ring A. (An algebra S over the commutative ring R is *separable* if S is a projective $S \otimes_R S$-module [8, p. 40].) We begin by showing that every such algebra B over A is also a Banach algebra, and that the induced mapping $X(B) \to X(A)$ on carrier spaces is a finite-fibered covering space projection. Thus separable algebras lead to covering spaces. Using Silov's idempotence theorem, we next show that the covering space mappings between any covering spaces of X(A) that are carrier spaces of separable algebras as above are induced from algebra homomorphisms. In other words, the functor $B \to X(B)$ above is full and faithful. For the functor to be a (contravariant) equivalance, we need to know that every covering space of X(A) comes from an algebra B. We show that this is the case if A is a regular Banach algebra without radical.

This equivalence, for the case of full rings of complex-valued continuous functions on compact spaces, was established by B. Wajnryb [13] and L. Childs [7]; since such function algebras are regular and without radical, this theorem is a consequence of our results here. Child's proof used the fact that rings of germs of continuous functions at a point are Henselian [7, Lemma, p. 32]. Since such rings are the local rings at maximal ideals of full function rings, the question arises whether the local rings at maximal ideals of an arbitrary Banach algebra are Henselian. We show that this is the case for regular Banach algebras without radical, but that there are examples to show that this is in general false.

We adopt the following conventions: $\mathbb{C}$ is the complex field, and all the rings we consider are commutative $\mathbb{C}$-algebras with identity. We use X(A) for the carrier space of the Banach algebra A, in the usual topology, although occasionally we also use the hull-kernel topology on X(A). If A is a Banach algebra and $a \in A$, we let $\hat{a}: X(A) \to \mathbb{C}$ denote the Gelfand transform of a.

For our purposes, the particular norm on a Banach algebra is not important, since we are concerned primarily with carrier spaces, and these do depend not on the norm chosen for the algebra, but only on the fact that the algebra is complete in some norm. Thus we refer throughout to *Banachable* algebras, by which we mean an algebra A such that there is some norm on A making A a Banach algebra.

If A is a Banachable algebra and $f \in A$, let $U_f = \{x \in X(A): \hat{f}(x) \neq 0\}$.

Our first goal is to show that a finitely generated projective faithful extension algebra of a Banachable algebra is also Banachable. We begin with some standard facts about Banach modules.

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