# FOLIATIONS AND LOCALLY FREE TRANSFORMATION GROUPS OF CODIMENSION TWO

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## INTRODUCTION

Let M be a manifold with a smooth foliation $\mathscr{F}$ of codimension q. Let E be the bundle of tangents to the leaves, and let $Q = T(M)/E$ be the normal bundle. Imbed Q as $E^{\perp}$ in T(M) once and for all via any Riemannian metric on M.

There is a fairly standard notion (see [1], for example) of a transverse H-structure for $\mathscr{F}$, where H is a Lie subgroup of the group $GL_q$ of $q \times q$ real nonsingular matrices. This is an H-reduction of Q that is invariant under the natural parallelism along leaves.

In the case where H is the group $G_k$ of all matrices of the form

$$\begin{bmatrix} I_k & 0 \\ A & B \end{bmatrix}$$

where $B \in GL_{q-k}$, the existence of a transverse $G_k$-structure means that some normal k-frame field $(Y_1, \cdots, Y_k)$ is invariant under the linear holonomy of each leaf. Equivalently [1, Corollary 1.5], we require $[Y_i, \Gamma(E)] \subset \Gamma(E)$ for $i = i, \cdots, k$. Letting $V = \text{span}_R \{Y_1, \cdots, Y_k\}$, we say that the $G_k$-structure is complete if each $Z \in V$ is a complete vector field on M (a condition that is automatic if M is compact).

*Definition.* $\rho(\mathscr{F})$ is the largest integer k for which $\mathscr{F}$ admits a complete transverse $G_k$-structure.

In particular, the statement $\rho(\mathscr{F}) = q$ means that $\mathscr{F}$ is a transversally complete e-foliation in the sense of [1], while the statement $\rho(\mathscr{F}) = 0$ means that $\mathscr{F}$ is not invariant under any nonsingular transverse flow on M.

In this paper we investigate the invariant $\rho(\mathscr{F})$ for the case $q = 2$, special applications being made to the situation in which the leaves of $\mathscr{F}$ are the orbits of a locally free Lie transformation group. It will be seen that this amounts to a generalization of results of E. Lima, H. Rosenberg, R. Sacksteder and S. P. Novikov on the rank and file of manifolds (see [5], [7], [8], [9], [11]).

Our basic result is Theorem 1. The term "vanishing cycle" which appears in that theorem, by now standard for foliations of codimension one ([3], [6], [1, Section 6]), is defined for higher codimension in a fairly obvious way in Section 1.

THEOREM 1. *Let* M *be closed and connected,* $\text{codim}(\mathscr{F}) = 2$, *and suppose that* $\mathscr{F}$ *admits no vanishing cycle. Then the condition* $\rho(\mathscr{F}) \geq 1$ *implies that* $\pi_1(M)$ *is infinite, and that if it is abelian it has rank at least* 2.

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