LINK MANIFOLDS

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INTRODUCTION

A *link-manifold* M^{2n+1} is a smooth closed manifold admitting a smooth action of the orthogonal group O(n) such that the isotropy subgroups are conjugate to O(n), O(n-1), or O(n-2), and such that for $n > 1$ the orbit space is the 4-disk D^4. The set of fixed points in M corresponds to a link $L \subset S^3 = \partial D^4$. (For $n > 1$, one assumes that all three orbit types occur. For n = 1, the orbit space is taken to be S^3 and the orbits are 0-spheres and fixed points.)

These manifolds occur readily in nature. For example, let $M_{a,b}$ denote the Brieskorn manifold [2] $V(Z_0^a + Z_1^b + Z_2^2 + \cdots + Z_{n+1}^2) \cap S^{2n+3}$. Then O(n) acts on $M_{a,b}$ via the last n coordinates, giving it the structure of a link manifold whose fixed-point set is a torus link of type (a, b).

In this paper, we generalize results of F. Hirzebruch and D. Erle [6] (see also [1] and [7] to [10]) to obtain a classification of link manifolds in terms of embedding invariants of links in S^3 (Theorems 10 and 11).

Link manifolds are a larger class than knot manifolds. We show that for $n = 2k - 1$ $(k \geq 2)$ *every* (n - 1) connected (2n + 1)-manifold that bounds a parallelizable manifold is a link manifold (Theorem 7).

The results in this paper were announced in [11].

1. LINKING NUMBERS AND INVARIANTS OF LINKS

A. *Seifert Pairing*

Given a Link $L \subset S^3$ with preassigned orientations for the components, one may form a connected oriented surface $F \subset S^3$ with $\partial F = L$ such that F induces the chosen orientation for L (see [16, p. 572]). Define

$$\theta: H_1(F) \times H_1(F) \to \mathbb{Z}$$

via $\theta(a, b) = \ell(i_* a, b)$, where $\ell(\ ,\ )$ denotes linking numbers in S^3 and i_* denotes the operation of pushing away from F in the positive normal direction. This bilinear pairing is called the Seifert pairing. Symmetrizing, one obtains the mapping $f: H_1(F) \times H_1(F) \to \mathbb{Z}$ defined by the formula $f(x, y) = \theta(x, y) + \theta(y, x)$.

An argument due to J. Levine [13] shows that if F' is another surface in S^3 whose boundary is ambient-isotopic to L and if V and V' denote matrices for the Seifert pairings for F and F', respectively, then V and V' are *related.* This means that V' may be obtained from V by a chain of operations of the two types

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