THERE EXIST NONREFLEXIVE INFLATIONS

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1. INTRODUCTION

Let H be a complex Hilbert space, and let B(H) be the algebra of bounded linear operators on H. If U is a subalgebra of B(H), then Lat U represents the set of closed subspaces of H invariant under every member of U. If F is any set of closed subspaces of H, then Alg F is the algebra of bounded linear operators that leave invariant every member of F.

It is obvious that if U is a weakly closed subalgebra of B(H), and if it contains the identity operator, then $U \subseteq \text{Alg Lat } U$.

Following P. R. Halmos, we say U is *reflexive* if U = Alg Lat U. Sufficient conditions for an algebra of operators to be reflexive were given in [9], [4], [6], [1], and other papers. Most results are obtained by means of techniques developed by W. B. Arveson [2] and D. E. Sarason [9]. It should be pointed out that the problem of classifying all reflexive algebras includes various generalizations of the invariant-subspace problem (see [7], for example).

An algebra of operators $\mathscr{S}$ on H is an n-*inflation* if there exist a Hilbert space K, a subalgebra $U \subset B(K)$, and an integer n $(1 \leq n < \infty)$ such that

$$H = \sum_{i=1}^{n} \oplus K \quad \text{and} \quad \mathscr{S} = U^{(n)} = \left\{ \sum_{i=1}^{n} \oplus Ai \text{ with } Ai = A \in U \right\}.$$

In [8], P. Rosenthal raised the question whether every 2-inflation is reflexive. In this paper, we show that there exist 2-inflations on an infinite-dimensional Hilbert space that are not reflexive. For algebras generated by more than one operator, the answer is still unknown even in the finite-dimensional case.

I would like to thank my teacher Professor Peter Rosenthal for many valuable discussions with respect to the results of this paper. The techniques used in the proof of Theorem 1 were discovered by him and H. Radjavi [7].

2. PRELIMINARIES

By an *operator algebra* we shall mean a weakly closed subalgebra of B(H) that contains the identity operator.

Let U be an operator algebra on H. If U is reflexive, then so is $U^{(2)}$. For suppose C is an operator on $H^{(2)}$ such that $\text{Lat } U^{(2)} \subset \text{Lat } C$. Since $H \oplus \{0\}$, $\{0\} \oplus H$, and $\{\langle x, x\rangle : x \in H\}$ are all in $\text{Lat } U^{(2)}$, it follows that $C = B \oplus B$ for some B on H. Therefore $\text{Lat } U^{(2)} \subset \text{Lat } B^{(2)}$ implies $\text{Lat } U \subset \text{Lat } B$, and, since U is reflexive, $B \in U$.

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