PENETRATION INDICES OF ARC-AND-BALL PAIRS AND UNCOUNTABLY MANY QUASI-TRANSLATIONS OF THE 3-SPHERE

Shin'ichi Kinoshita and Carl D. Sikkema

INTRODUCTION

This paper is a continuation of [4], and we assume that readers are familiar with that paper. We shall prove, among other things, that there exist uncountably many mutually inequivalent quasi-translations of the 3-sphere. From this it follows that *there exist uncountably many open, orientable* 3-*manifolds whose fundamental group is infinite cyclic and whose universal covering space is* 3-*space* E^3.

In addition to the statement above, which can be proved rather simply, we study some related topics. Not only do they have independent interest, but they are helpful for understanding the background of the proofs of the statement above and the whole structure of the study developed in [4] and this paper.

The statement above was announced in [4]. There we also announced that the composition of two open arc-and-ball pairs (see below) is not commutative. However, we have found a gap in the proof.

In Section I we study an *open arc-and-ball pair* (a, B), where a is an arc in a 3-ball B such that only the endpoint p(a) of a is on the boundary ∂B of B and a is locally tame in B except at p(a). (This is what in [4] we called an arc-and-ball pair.) We introduce the penetration index P(a, B) for (a, B) and prove that

$$P((a_1, B_1) \# (a_2, B_2)) = P(a_1, B_1) \cdot P(a_2, B_2).$$

In Section II we study a *closed arc-and-ball pair* [a, B], where a is an arc in a 3-ball B such that only the initial point q(a) and the endpoint p(a) are on ∂B, and such that a is locally tame in B except possibly at p(a). One of the significant differences between the two concepts (a, B) and [a, B] is that the infinite composition $\#_{n=-\infty}^{+\infty} [a_n, B_n]$ can be defined, but $\#_{n=-\infty}^{+\infty} (a_n, B_n)$ can not. We shall apply this infinite composition in Section III.

In [4], we associate with each (a, B) a quasi-translation h(a, B) of the 3-sphere. In Section III, we consider open arc-and-ball pairs that are constructed from Wilder arcs and their associated quasi-translations. We study the mutual inequivalence of some of these quasi-translations through the concept of positively characteristic translation curves, and we close the discussion with the classification of Wilder arcs by R. H. Fox and O. G. Harrold [3].

Received April 5, 1972.
The authors of this paper are partially supported by NSF Grant GP-19964.

Michigan Math. J. 20 (1973).