SENSE-PRESERVING PL INVOLUTIONS OF SOME LENS SPACES

K. W. Kwun

1. INTRODUCTION

Let $L = L(p, q)$ be a 3-dimensional lens space. We say that L is *symmetric* if $q^2 \equiv \pm 1 \pmod{p}$. Recall that $L(p, q)$ and $L(p, q')$ are homeomorphic [3], [4] if and only if $q' \equiv \pm q$ or $qq' \equiv \pm 1 \pmod{p}$. Hence, symmetry of L is a topological property. A map $f: L \to L$ is called *sense-preserving* if f induces the identity of $H_1(L)$. For odd indices p ($p \geq 3$), we investigate all PL involutions of L that preserve sense and have nonempty fixed-point sets. It is known [1] that if $p \geq 3$, then L does not have an orientation-reversing involution. Also, simple examples show that an orientation-preserving involution need not be sense-preserving. If an involution can be extended to an effective circle action, it must clearly be sense-preserving. In our case, it turns out that the condition is also sufficient.

THEOREM. *Let* $L = L(p, q)$ (p *odd*, $p \geq 3$). *Let* h *be a* PL *involution of* L *with a nonempty fixed-point set. The* Z_2*-action generated by* h *can be extended to an effective* S^1*-action if and only if* h *is sense-perserving. Up to* PL *equivalences, there is exactly one such sense-preserving involution* h *if* L *is symmetric, and there are exactly two if* L *is not symmetric.*

Henceforth, we assume that $L = L(p, q)$ (p odd, $p \geq 3$) and that h is a sense-preserving PL involution of L with nonempty fixed-point set F. We shall simply call the orbit space of the Z_2-action generated by h the *orbit space* of h.

Remark. We can easily describe the orbit space of h as follows. If $q^2 \equiv \pm 1 \pmod{p}$, the orbit space is $L(p, q')$, where q' is any integer such that $2q' \equiv q \pmod{p}$. If $q^2 \not\equiv \pm 1$, we have two nonhomeomorphic orbit spaces $L(p, q')$ and $L(p, q'')$, where q' and q'' are any integers such that $2q' \equiv q$ and $2qq'' \equiv 1 \pmod{p}$.

2. THE FIXED-POINT SET F OF h

PROPOSITION 2.1. *F is a simple closed curve.*

Proof. Since L is a Z_2-homology sphere, F must be a sphere. Since $F \neq \emptyset$ and h preserves orientation, F is a simple closed curve, by the parity theorem.

PROPOSITION 2.2. *Let* $i: F \subset L$. *Then*

$$i_{\#}: \pi_1(F) \to \pi_1(L)$$

is an epimorphism.

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