NONFOLDING MAPS AND THE SINGULAR-REGULAR-NEIGHBORHOOD THEOREM

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1. INTRODUCTION

Suppose K is a finite topological complex in a 3-manifold M, and f is a map of M into a 3-manifold N such that f(K) is a topological complex and $K = f^{-1}f(K)$. It is known that f(K) may be tame even though K is not tame [6]. If K is tame and f is a homeomorphism on $M^3 - K$, then f(K) is tame [13]. We show that if K is tame and f is a homeomorphism on K and f "doesn't fold" at any point of f(K), then f(K) is tame (Theorem 4). This is an extension of the singular-regular-neighborhood theorem that was established for surfaces by J. Hempel [12, Theorem 2] and for finite graphs by J. W. Cannon [8]. S. Armentrout showed in [2] that if K is tame and if f is onto N and defines a cellular upper-semicontinuous decomposition of M none of whose nondegenerate elements meets K, then f(K) is tame. Corollary 2 implies that "cellular" may be replaced by "monotone." Theorem 5 is more general than Theorem 4 and Corollary 2; it deals with nonfolding maps that are not necessarily homeomorphisms on either K or $M^3 - K$.

2. NOTATION AND TERMINOLOGY

The terms n-*manifold, triangulation, polyhedron,* and *tamely embedded* are used as in [5]. A subset A of an n-manifold M is *cellular* in M if and only if there exists a sequence $C_1, C_2, \cdots$ of n-cells in M such that

(1) for each positive integer i, $C_{i+1} \subset \operatorname{Int} C_i$, and

(2) $\bigcap_{i=1}^{\infty} C_i = A$.

Suppose f is a map of an n-manifold M into an n-manifold N, $p \in N$, and $f^{-1}(p)$ is cellular in M. Choose n-cells B and C in M and N, respectively, such that

$$p \in \operatorname{Int} C, \qquad f^{-1}(p) \subset \operatorname{Int} B, \qquad f(B) \subset \operatorname{Int} C.$$

We say f *folds* (*doesn't fold*) *at* p if and only if $f \mid \operatorname{Bd} B$ is homotopically trivial (nontrivial) in C - p. Note that this definition is independent of the choice of n-cells B and C.

Suppose A is a subset of an n-manifold N and f is a map of an n-manifold M into N such that $A \subset f(M)$. We let G_A denote the decomposition of M whose elements are the inverse images of points in A and the singletons of $M - f^{-1}(A)$.

A *topological complex* is a space homeomorphic to a locally finite simplicial complex. Let K be a connected topological complex in a 3-manifold N. We say K has a *singular-regular neighborhood* if there exist a triangulated 3-manifold M, a

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