PAIRS OF ADDITIVE EQUATIONS

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1. INTRODUCTION

Let

(1) $$F = \sum_{i=1}^{n} a_i x_i^3 \quad \text{and} \quad G = \sum_{i=1}^{n} b_i x_i^3,$$

where the coefficients are integers. We consider the equations

(2) $$F = G = 0.$$

H. Davenport and D. J. Lewis [2] proved that if $n \geq 16$, there exists a nontrivial solution of (2) in every p-adic field, and that if $n \geq 18$ there exists a nontrivial solution of (2) in rational integers. By applying an appropriate form of Hua's Lemma, we simplify the analytic part of the argument (particularly the treatment of the minor intervals), and we obtain the following stronger result.

THEOREM 1. *If* $n \geq 17$, *then the equations* (2) *have a nontrivial solution in rational integers.*

More generally, we can consider pairs of additive equations

(3) $$f = \sum_{i=1}^{n} a_i x_i^k = 0, \quad g = \sum_{i=1}^{n} b_i x_i^k = 0,$$

where $k \geq 3$ and the coefficients are integers.

THEOREM 2. *If*

(i) *the equations* (3) *have a nonsingular solution in every* p-*adic field and in the real field, if*

(ii) $n > 2^{k+1}$,

and if, in case the degree k *of the equations* (3) *is even,*

(iii) *every member of the pencil* $\{\lambda f + \mu g\}$ $[(\lambda, \mu) \neq (0, 0)]$ *contains at least* $2^k + 1$ *variables with nonzero coefficients,*

then the equations (3) *have a nontrivial solution in rational integers.*

Theorem 2 will not be proved; it can be proved in precisely the same way as Theorem 1. From the p-adic results of Davenport and Lewis [3], we shall deduce the following result, which is new for $k < 12$ but inferior to results of Davenport and Lewis [4] for large k.

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