TWO EXAMPLES IN SURFACE AREA THEORY

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1. INTRODUCTION

By a k-surface in R^k, we mean the class of Fréchet-equivalent, continuous mappings $f\colon X \to R^k$ from a compact topological k-cell X in R^k $(k \geq 2)$. We investigate representation problems for such k-surfaces of finite Lebesgue k-area. In particular, we examine the following two notions.

Absolutely continuous mappings. A continuous mapping $f\colon X \to R^k$ $(X \subset R^k)$ is said to be *absolutely continuous* (briefly, AC) if there exists a Lebesgue-integrable function ϕ on int X such that $L(f, G) = \int_G \phi(x)\,dx$ for every open subset G of X. Here, L(f, G) denotes the Lebesgue k-area of the restriction of f to G.

Differentiably absolutely continuous mappings. A continuous mapping $f\colon X \to R^k$ $(X \subset R^k)$ is said to be *differentiably absolutely continuous* (DAC) if it is AC and possesses a weak total differential a.e. in int X. (See [7].)

Equivalent definitions of absolute continuity have been used in [2] for $k = 2$, in [1] for $k > 2$, and in [7] for $k \geq 2$. If f is AC, then we may take $\phi = |J|$, where J is the generalized Jacobian of f. If f is DAC, then we may take $\phi = |j|$, where j is the ordinary Jacobian of f.

By means of two examples of three-dimensional Fréchet surfaces of finite Lebesgue 3-area, we show that

(1) finiteness of 3-area of a Fréchet surface does not imply the existence of an absolutely continuous representation, and

(2) there exists a Fréchet surface of finite 3-area with an absolutely continuous representation but no differentiabily absolutely continuous representation.

We use the surface discussed in [6] in the first example, and a surface of the type discussed in [3] in the second example.

It is known that for two-dimensional Fréchet surfaces, such examples never exist. (See [2].)

2. PRELIMINARIES

For use in the examples below, we recall the construction of some multiplicity functions and k-areas associated with Lebesgue k-area.

O(y, f, I) denotes the usual topological index of a point y in R^k with respect to the restriction of f to a polyhedral region I contained in X. Corresponding to each subset A of X, we define the *essential multiplicity*

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