THE INDEX OF A SUBGROUP OF THE SYMPLECTIC MODULAR GROUP

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1. INTRODUCTION

Let Ω_n be the semigroup of all n-by-n matrices with rational integral entries, and let $\mathcal{M}_n$ denote the symplectic modular group of degree n; that is, let $\mathcal{M}_n$ be the group of all matrices $M \in \Omega_{2n}$ that satisfy the equation

$$M'JM = J, \tag{1}$$

where $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$, I being the identity n-by-n matrix. If $M \in \mathcal{M}_n$ is partitioned as $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ with $A, B, C, D \in \Omega_n$, it is easy to see that (1) is equivalent to the conditions

$$AB' = BA', \quad CD' = DC', \quad \text{and} \quad AD' - BC' = I. \tag{2}$$

A matrix $N \in \Omega_{2n}$ is called m-*symplectic* (m a positive integer) if it satisfies the condition

$$N'JN = mJ. \tag{3}$$

Denote the set of m-symplectic matrices by $\mathcal{M}_n(m)$, and call two matrices $M, N \in \mathcal{M}_n(m)$ *left-associated* if there exists an $M_1 \in \mathcal{M}_n$ such that $M = M_1 N$, and *equivalent* if there exist $M_2, M_3 \in \mathcal{M}_n$ such that $M = M_2 N M_3$. Clearly, the relations of being left-associated and of being equivalent are equivalence relations on $\mathcal{M}_n(m)$. In [5], the following two results were proved.

THEOREM 1. *An* m-*symplectic matrix is left-associated to exactly one matrix of the form*

$$\begin{bmatrix} Q_1 & m^{-1}SQ \\ 0 & Q_2 \end{bmatrix},$$

where $Q_1, Q_2, S \in \Omega_n$, Q_1 *is in Hermite normal form,* $\det Q_1 > 0$, $Q_1 Q_2' = mI$, $S = [s_{ij}]$ *is symmetric,* $0 \leq s_{ij} < m$ $(1 \leq i, j \leq n)$, *and* $SQ_2 \equiv 0 \pmod{m}$.

The Hermite normal form of a matrix in Ω_n is the unique form to which it can be reduced by premultiplication by a suitable $U \in \Omega_n$ with determinant unity. For a more detailed explanation, see [2, p. 32].

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