A REMARK ON HOMOTOPY AND CATEGORY DOMINATION

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M. Mather [2] showed that the set of homotopy types of spaces dominated by (finite) polyhedra is countable (see also [1]). The purpose of this note is to give a simpler proof of a more general result.

Given a category C, we say that a morphism $f: X \to X$ is an *idempotent* if $f \circ f = f$. By $I(X)$ we denote the set of all idempotents of X.

We say X *dominates* Y provided there exists a *domination* (h, g), that is, a pair $(h: X \to Y,\ g: Y \to X)$ such that $h \circ g = 1_Y$. If (h, g) is a domination, then $g \circ h \in I(X)$, and we say that $g \circ h$ is the idempotent associated with (h, g).

THEOREM 1. *If* X *dominates* Y_i *with domination* (h_i, g_i), *for* $i = 1, 2$, *and if the two associated idempotents are the same, then* Y_1 *is isomorphic to* Y_2.

Proof. The morphisms $h_{3-i} \circ g_i: Y_i \to Y_{3-i}$ $(i = 1, 2)$ are isomorphisms, each inverse to the other:

$$(h_i \circ g_{3-i}) \circ (h_{3-i} \circ g_i) = h_i \circ (g_{3-i} \circ h_{3-i}) \circ g_i = h_i \circ (g_i \circ h_i) \circ g_i = 1_{Y_i} \qquad (i = 1, 2).$$

COROLLARY. *The class of isomorphism classes dominated by* X *has cardinality at most* card I(X).

Application. Up to homeomorphism, there are only a countable number of polyhedral pairs and of homotopy classes of mappings of a polyhedral pair into itself; hence, the Corollary yields the following result.

THEOREM 2. *The class of homotopy types of topological pairs dominated by polyhedral pairs is countable.*

REFERENCES

1. J. M. Kister, *Homotopy types of* ANR's. Proc. Amer. Math. Soc. 19 (1968), 195.

2. M. Mather, *Counting homotopy types of manifolds.* Topology 4 (1965), 93-94.

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Received April 13, 1971.
This paper was written with support from the National Science Foundation.

Michigan Math. J. 18 (1971).