MEROMORPHIC FUNCTIONS OF BOUNDED BOUNDARY ROTATION

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1. INTRODUCTION

For $k \geq 2$, let Λ_k denote the class of functions f, given by

$$f(z) = \frac{1}{z} + a_0 + a_1 z + \cdots, \tag{1.1}$$

that are analytic in $U = \{z: |z| < 1\}$ except for a simple pole at $z = 0$ and have an integral representation of the form

$$f'(z) = -\frac{1}{z^2} \exp\left\{\int_0^{2\pi} \log(1 - ze^{-it})\, dm(t)\right\}, \tag{1.2}$$

where m is a real-valued function of bounded variation on $[0, 2\pi]$ satisfying the conditions

$$\int_0^{2\pi} dm(t) = 2, \qquad \int_0^{2\pi} |dm(t)| \leq k, \qquad \int_0^{2\pi} e^{-it} dm(t) = 0. \tag{1.3}$$

Simple calculations show that the third of conditions (1.3) guarantees that the singularity of f at $z = 0$ is a simple pole with no logarithmic term.

The class Λ_k was introduced by J. Pfaltzgraff and B. Pinchuk in [5], where they showed that $f \in \Lambda_k$ if and only if f maps the unit disc onto a domain containing infinity, with boundary rotation at most $k\pi$ (for a definition of this concept, see [4]). Hence the union of the classes Λ_k is called the family of meromorphic functions of bounded boundary rotation.

Let V_k denote the class of functions g, given by

$$g(z) = z + b_2 z^2 + \cdots,$$

that are analytic in U, satisfy the condition $g'(z) \neq 0$ in U, and map U onto a domain with boundary rotation at most $k\pi$. V. Paatero [4] showed that $g \in V_k$ if and only if

$$g'(z) = \exp\left\{\int_0^{2\pi} \log(1 - ze^{-it})^{-1} dm(t)\right\}, \tag{1.4}$$

Received July 31, 1970.

The author was supported by an NDEA Graduate Fellowship at the University of Maryland.

Michigan Math. J. 18 (1971).