THE QUALITATIVE BEHAVIOR OF THE SOLUTIONS OF A NONLINEAR VOLTERRA EQUATION

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1. INTRODUCTION

In this paper, we consider the equation

$$(1.1) \qquad x'(t) = \int_0^t b(t-\tau)g(x(\tau))\,d\tau + f(t) \qquad (0 \leq t < \infty),$$

where $x(0)$ is a prescribed real number and $b(t)$, $f(t)$, $g(x)$ are prescribed real functions. The following is our main result.

THEOREM 1. *Let*

$$(1.2) \qquad b(t) \in L_1(0, 1),$$

$$(1.3) \qquad [-1]^k b^{(k)}(t) \leq 0 \qquad (0 < t < \infty;\ k = 0, 1, 2),$$

$$(1.4) \qquad b(t) \not\equiv b(0+),$$

$$(1.5) \qquad g(x) \in C(-\infty, \infty),$$

$$(1.6) \qquad f(t) \in C[0, \infty) \cap L_1[0, \infty),$$

and let $x(t)$ *be a solution of* (1.1) *on* $0 \leq t < \infty$ *such that*

$$(1.7) \qquad \sup_{0 \leq t < \infty} |x(t)| < \infty .$$

Then $\lim_{t\to\infty} g(x(t))$ *exists and*

$$(1.8) \qquad \lim_{t\to\infty} g(x(t)) = 0 .$$

If, in addition,

$$(1.9) \qquad \lim_{t\to\infty} f(t) = 0,$$

then $\lim_{t\to\infty} x'(t) = 0$.

In (1.3), we assume that $b''(t)$ exists and is finite on $0 < t < \infty$. Theorem 1 obviously remains true if (1.3) is replaced by

$$[-1]^k b^{(k)}(t) \geq 0 \qquad (0 < t < \infty;\ k = 0, 1, 2).$$

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