UNIFORM CONVERGENCE OF FOURIER SERIES ON GROUPS, I

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In 1940, R. Salem [6] proved that the Fourier series of a continuous periodic function converges uniformly if $\lim_{n\to\infty} T_n(x) = \lim_{n\to\infty} Q_n(x) = 0$ uniformly in x, where

$$T_n(x) = \sum_{p=0}^{(n-1)/2} (p+1)^{-1}[f(x + 2p\pi/n) - f(x + (2p+1)\pi/n)],$$

and where $Q_n(x)$ is obtained from $T_n(x)$ by changing π into $-\pi$. In a recent paper [5], C. W. Onneweer proved a similar theorem for Walsh-Fourier series. In this paper, we extend this result to continuous functions defined on certain compact, 0-dimensional, metrizable, abelian groups. Such groups and their character groups were first studied by N. Ja. Vilenkin [7]. The significance of our result is evidenced primarily by its corollaries, as was the case with Salem's original theorem.

1. THE GROUPS G AND X

Let G be a compact, 0-dimensional, metrizable, abelian group, and let X be its character group. Then X is a discrete, countable, abelian torsion group [4, Theorems 24.15 and 24.26]. Vilenkin [7, Sections 1.1 and 1.2] established the existence of an increasing sequence of finite subgroups $\{X_n\}$ of X such that

(i) $X_0 = \{\chi_0\}$, where χ_0 is the identity character on G,

(ii) each X_n/X_{n-1} is of prime order p_n, and

(iii) $X = \bigcup_{n=0}^{\infty} X_n$.

Moreover, the subgroups X_n can be chosen so that there exists a sequence $\{\phi_n\}$ of elements of X satisfying the conditions

$$\text{(i) } \phi_n \in X_{n+1} \setminus X_n \quad \text{and} \quad \text{(ii) } \phi_n^{p_{n+1}} \in X_n.$$

Using these ϕ_n, we can enumerate the elements of X as follows. Let $m_0 = 1$ and $m_n = \prod_{i=1}^{n} p_i$. Each natural number k can be represented uniquely as $k = \sum_{i=0}^{s} a_i m_i$, with $0 \leq a_i < p_{i+1}$ for $0 \leq i \leq s$; we define χ_k by the formula $\chi_k = \phi_0^{a_0} \cdot \ldots \cdot \phi_s^{a_s}$. Then $X_n = \{\chi_i \mid 0 \leq i < m_n\}$.

Next, let G_n be the annihilator of X_n, that is, let

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