QUASI-p-REGULARITY OF SYMMETRIC SPACES

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INTRODUCTION

If X and Y are CW-complexes, we say that Y is p-*equivalent* to X (notation: $X \underset{p}{\simeq} Y$) if there exists a map $f: X \to Y$ such that

$$f^*: H^*(Y; Z_p) \cong H^*(X; Z_p).$$

Following [10], we say that X is p-*regular* if it is p-equivalent to a product of spheres. We call X *quasi*-p-*regular* if X is p-equivalent to a product of spheres and spaces $B_n(p)$ satisfying the condition

$$H^*(B_n(p); Z_p) \cong \Lambda(x_{2n+1}, \mathfrak{P}^1 x_{2n+1}).$$

In [7], P. G. Kumpel discussed the p-*regularity* of irreducible symmetric spaces. The purpose of this paper is to extend the study to the *quasi*-p-*regularity* of irredicible symmetric spaces.

Let G be a compact, connected, simply connected Lie group with an involution $\sigma: G \to G$. Let K be the identity component of the fixed-point set of σ, and assume that K is totally nonhomologous to zero in G with real coefficients. Then the irreducible symmetric spaces G/K satisfying the hypotheses above are

(i) $(K \times K)/K$,

(ii) $SU(2n + 1)/SO(2n + 1)$,

(iii) $SU(2n)/Sp(n)$,

(iv) $Spin(2n)/Spin(2n - 1)$,

(v) E_6/F_4.

As is well known, $(K \times K)/K$ is isomorphic to K. The quasi-p-regularity of the Lie groups was discussed in [8]. Since

$$Spin(2n)/Spin(2n - 1) = S^{2n-1},$$

the space (iv) is quasi-p-regular. Therefore it is sufficient to study the quasi-p-regularity of (ii), (iii), and (v). Our results (Theorems 4.2, 4.3, and 4.4) are as follows.

$SU(2n)/Sp(n)$ *is quasi*-p-*regular if and only if* $p \geq n$.

$SU(2n + 1)/SO(2n + 1)$ *is quasi*-p-*regular if and only if* $p \geq n + 1$.

E_6/F_4 *is quasi*-p-*regular if and only if* $p \geq 5$.

Corollary 4.5 answers negatively a question of Kumpel [7].

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