SOME INTERPOLATION PROBLEMS IN HILBERT SPACES

Bruce L. Chalmers

INTRODUCTION

In 1961, H. S. Shapiro and A. L. Shields [9] investigated interpolation problems in several function spaces. The present paper is an extension of the part of their work that treats weighted interpolation (by pointwise evaluation at a sequence of points) in several classical Hilbert spaces, especially H_2. First we shall obtain results concerning interpolation by sequences of arbitrary continuous linear functionals in an arbitrary Hilbert space, and later we shall obtain more specialized results involving interpolation by evaluation of derivatives in classical Hilbert spaces.

Let $\{z_i\}$ denote a sequence of points in the disk $D = [|z| < 1]$. Then $\{z_i\}$ is called a *Carleson sequence* if

$$\prod_{i \neq j} |(z_i - z_j)/(1 - \bar{z}_j z_i)| \geq \delta > 0 \qquad (j = 1, 2, \cdots).$$

The sequence $\{z_i\}$ is called an *exponential sequence* if

$$(1 - |z_{j+1}|)/(1 - |z_j|) \leq r < 1 \qquad (j = 1, 2, \cdots),$$

and $\{z_i\}$ is called a *radial sequence* if all the z_i lie on one radius. An exponential sequence is a Carleson sequence, and a radial Carleson sequence is an exponential sequence. Let $\mathscr{L}^1, \mathscr{L}^2, \cdots$ denote a sequence of continuous linear functionals on a Hilbert space H, let $\hat{\mathscr{L}}^i$ denote the functional $\mathscr{L}^i$ divided by its norm, and let $\hat{T}f = \{\hat{\mathscr{L}}^i f\}_{i=1}^{\infty}$. Shapiro and Shields [9] showed that if $\mathscr{L}^i$ is pointwise evaluation at z_i on the Hardy space H_2, then $\hat{T}(H_2) = \ell_2$ if and only if $\{z_i\}$ is a Carleson sequence.

In Section 2, we generalize the notions of Carleson sequence and exponential sequence and define the notion of projective sequence (which includes the radial sequences in the case of pointwise evaluation in H_2) in an arbitrary Hilbert space. We then show that if $\mathscr{L}^1, \mathscr{L}^2, \cdots$ is a sequence in the dual of the Hilbert space H, then

(i) the relation $\hat{T}(H) = \ell_2$ implies that $\{\mathscr{L}^i\}$ is a Carleson sequence;

(ii) if $\{\mathscr{L}^i\}$ is an exponential sequence, then $\hat{T}(H) \subset \ell_2$;

(iii) if $\{\mathscr{L}^i\}$ is an exponential sequence with a certain restriction, then $\hat{T}(H) \supset \ell_2$;

(iv) a projective Carleson sequence is an exponential sequence (see Theorems 2.7, 2.8, 2.9, 2.12, and Corollary 2.13).

In Note 2.14 we indicate that in general these results cannot be improved.

Received October 28, 1969.

Michigan Math. J. 18 (1971).