AN ISOMORPHIC CHARACTERIZATION OF L_p AND c_0-SPACES. II.

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In a previous paper of the same title [14], we have proved that a Banach space X is isomorphic either to c_0 or to an L_p-space $(1 \leq p < \infty)$ over a finite measure space if and only if it is a cyclic space $X = \text{clm}\{Px_0 \mid P \in \mathscr{B}\}$ $(x_0 \in X)$ relative to a σ-complete Boolean algebra $\mathscr{B}$ of projections that has a *two-sided estimate.* The latter condition is to be interpreted in the sense that there exist a constant K and a function ψ (defined in the space of sequences of complex numbers) such that the inequalities

$$K^{-1}\psi(\{\|P_n x\|\}) \leq \|x\| \leq K\psi(\{\|P_n x\|\})$$

hold for each $x \in X$ and for each sequence of disjoint projections $P_n \in \mathscr{B}$ whose sum is the identity I.

Other characterizations in terms of Boolean algebras of projections with two-sided estimates have been obtained in [8] and [10] for the $\mathscr{L}_p$-spaces introduced by J. Lindenstrauss and A. Pełczyński in [7].

In the present paper we weaken this condition: we show that instead of the two-sided estimate for $\mathscr{B}$ we need merely the existence of a similar function ϕ, with values in $[0, \infty]$, such that a series $\sum_{n=1}^{\infty} P_n x_n$ is weakly convergent (not necessarily to a vector in X) if and only if $\phi(\{\|P_n x_n\|\}) < \infty$ for each sequence $\{x_n\}$ $(x_n \in X)$ and each sequence of disjoint projections $P_n \in \mathscr{B}$. We shall use this result to prove our main theorem, which is another isomorphic characterization of c_0 and L_p, this time involving the existence of positive projections on every sublattice of a σ-Dedekind complete (conditionally σ-complete) Banach lattice. This theorem represents an isomorphic version of a recent result of T. Ando [1].

1. CYCLIC SPACES ISOMORPHIC TO L_p AND c_0

For the notions and the terminology used in this paper, we refer the reader to [14] (see also [2], [3], and [7]).

We begin by showing that a cyclic space having enough subspaces isomorphic to ℓ_p $(1 \leq p < \infty)$ is in fact isomorphic itself to an L_p-space for the same p.

PROPOSITION 1. *A Banach space* X *is isomorphic to an* L_p*-space* $(1 \leq p < \infty)$ *over a finite measure space if and only if it is a cyclic space*

$$X = \mathfrak{M}(x_0) = \text{clm}\{Ex_0 \mid E \in \mathcal{E}\}$$

relative to a σ*-complete Boolean algebra* $\mathcal{E}$ *of projections such that for each* $x \in X$ *and each infinite sequence of disjoint projections* $E_n \in \mathcal{E}$ $(E_n x \neq 0;\ n = 1, 2, \cdots)$ *the basis* $\{E_n x/\|E_n x\|\}$ *is equivalent to the natural basis of* ℓ_p.

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