NONLINEAR HYPERBOLIC PROBLEMS WITH SOLUTIONS ON PREASSIGNED SETS

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1. INTRODUCTION AND SUMMARY

This paper is concerned with initial-value problems and mixed boundary problems for hyperbolic systems of quasi-linear partial differential equations in two independent variables. The setting is classical: we shall assume that the coefficients of the equations and the data possess continuous first derivatives, and the solutions will be C^1-functions satisfying the equations everywhere on their domains of definition.

We consider the hyperbolic initial-value problem

$$z_t + A(x, t, z)z_x = f(x, t, z) \qquad ((x, t) \in \mathscr{R}(T)), \tag{1.1}$$

$$z(x, 0) = \phi(x) \qquad (x \in R), \tag{1.2}$$

where R is the set of real numbers, $\mathscr{R}(T)$ is the strip $R \times [0, T]$, the function $z = z(x, t)$ takes values in R_m (the real euclidean m-dimensional space), A is a matrix-valued function, and f, ϕ are vector-valued functions. Our main object is to consider the following problem.

Problem I. Find conditions on A and f that guarantee the existence of a class of initial conditions for each $T > 0$ such that for every ϕ in such a class, the Cauchy problem (1.1), (1.2) has a solution z on the strip $\mathscr{R}(T)$.

Results of this type are useful, in particular, in the control theory of hyperbolic equations; see [2].

It is known (see for instance P. Lax [9], P. Hartman and A. Wintner [7, p. 855], A. Douglis [6, p. 149], M. Cinquini-Cibrario and S. Cinquini [1, Chapter V]) that if (1.1) is hyperbolic and A, f, ϕ are prescribed C^1-functions that are bounded, together with their first derivatives, then there exist some real number $T > 0$ and a (unique) C^1-function $z = z(x, t)$ on $\mathscr{R}(T)$ satisfying (1.1), (1.2).

However, even if $\phi = 0$, the local solution z cannot be extended to an arbitrarily prescribed strip $\mathscr{R}(T')$ under the above hypotheses. We give a scalar example in Section 3 to illustrate this fact. On the other hand, we shall prove (Theorem 3.II) that if A, f satisfy the conditions above and if in addition $f_x = 0$, then for each $T' > 0$, there is a class of initial data (namely C^1-functions with sufficiently small first derivatives) such that if ϕ is chosen in that class, the local solution z can be extended to $\mathscr{R}(T')$.

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