MULTIPLIERS AND SETS OF UNIQUENESS OF L^p

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Let G be a nondiscrete, locally compact Abelian (LCA) group and Γ its character group. In this paper, we establish the existence of subsets of G of positive Haar measure that are sets of uniqueness for Fourier transforms of functions in $L^p(\Gamma)$ $(1 \leq p < 2)$. We draw from this result several consequences concerning multipliers of $L^p(\Gamma)$, and we establish related results for other function spaces. Specifically, in Section 1, we prove that there exist subsets $E \subset G$ of positive Haar measure with the property that no nonzero function whose Fourier transform belongs to

$$\bigcup_{1 \leq p < 2} L^p(\Gamma)$$

can be carried by E. This is an extension of a theorem of Y. Katznelson: Katznelson proved this result for the case where $G = T$ (the circle group) and $\Gamma = Z$ (the integers) (see [10], [11, p. 101]), and he has indicated to one of us that the proof of his theorem can be modified to give the same result for the real line. The proof presented here is in some sense independent of the algebraic structure of the group and, in fact, yields also an analogous result for orthogonal systems on a nonatomic measure space (see Remark 1.3 below). In Section 2, we use the results of Section 1 to extend to general nondiscrete LCA groups G several theorems concerning multipliers and Fourier transforms of functions in $L^p(\Gamma)$. These theorems were proved for special classes of groups by the authors (working independently) [3], [6], by R. E. Edwards, and by L. Hörmander [9].

In Section 3, we characterize the multipliers of the space $L^1_p(\Gamma)$ consisting of those functions in $L^1(\Gamma)$ whose Fourier transforms are in $L^p(G)$. Our result has been stated previously in [12]; the proof of this result given in [12] is, however, incorrect.

1. SETS OF UNIQUENESS FOR L^p

If G is a nondiscrete LCA group and Γ is its character group, we define a set of uniqueness for $L^p(\Gamma)$ $(1 \leq p < 2)$ as follows.

Definition. Let E be a measurable subset of G; then E is called a *set of uniqueness* for $L^p(\Gamma)$ if no nonzero integrable function f, carried by E, satisfies the condition $\hat{f} \in L^p(\Gamma)$.

(Here, as in the sequel, $\hat{f}$ denotes the Fourier transform of f.)

We can now state the main result of this section.

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